6 Topics in Analytic Geometry

6.1 Lines

Objectives

- Find the inclination of a line.
- Find the angle between two lines.
- Find the distance between a point and a line.

Inclination of a Line

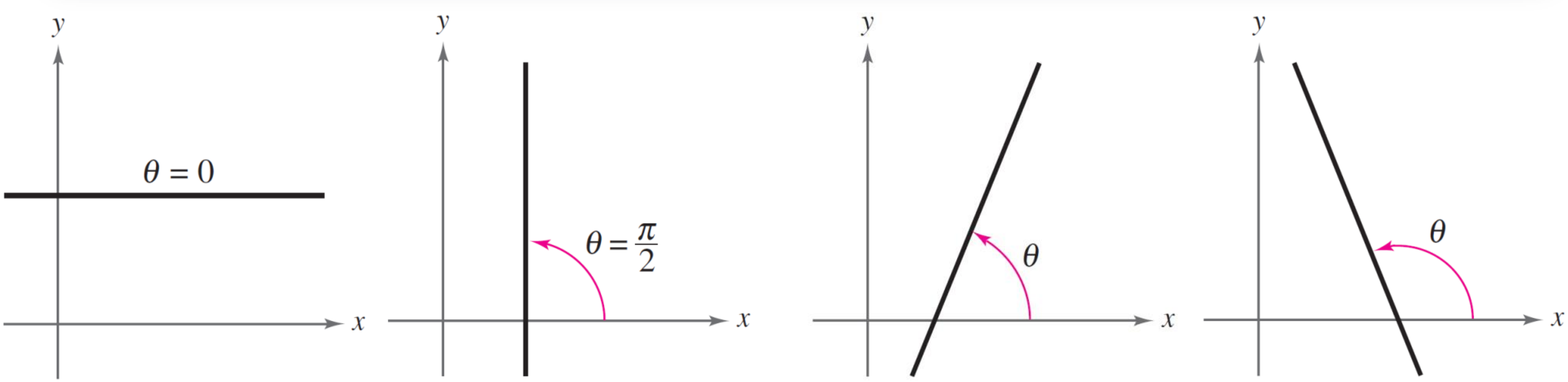
Inclination of a Line

In this section, you will look at the slope of a line in terms of the angle of inclination of the line.

Inclination of a Line

Every nonhorizontal line must intersect the *x*-axis. The angle formed by such an intersection determines the **inclination** of the line, as specified in the following definition.

> **Definition of Inclination**
>
> The **inclination** of a nonhorizontal line is the positive angle θ (less than π) measured counterclockwise from the x-axis to the line. (See figures below.)



Horizontal Line | Vertical Line | Acute Angle | Obtuse Angle

Inclination of a Line

The inclination of a line is related to its slope in the following manner.

Inclination and Slope

If a nonvertical line has inclination θ and slope m, then

$$m = \tan \theta.$$

Example 1 – *Finding the Inclination of a Line*

Find the inclination of (a) $x - y = 2$ and (b) $2x + 3y = 6$.

Solution:

a. The slope of this line is $m = 1$. So, its inclination is determined from $\tan \theta = 1$. Note that $m \geq 0$. This means that

$$\theta = \arctan 1 = \pi / 4 \text{ radian}$$

$$= 45°$$

as shown in Figure 6.1(a).

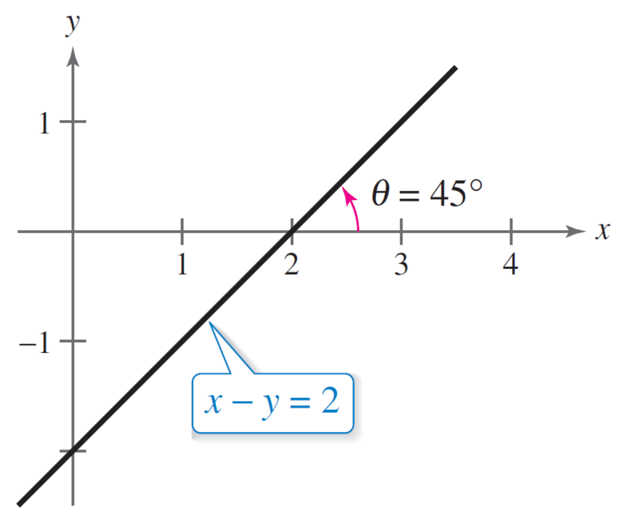


Figure 6.1(a)

Example 1 – *Solution* cont'd

b. The slope of this line is $m = -\frac{2}{3}$. So, its inclination is determined from $\tan\theta = -\frac{2}{3}$. Note that $m < 0$. This means that

$$\theta = \pi + \arctan\left(-\tfrac{2}{3}\right)$$

$$\approx \pi + (-0.5880)$$

$$\approx 2.5536 \text{ radians}$$

$$\approx 146.3^\circ$$

as shown in Figure 6.1(b).

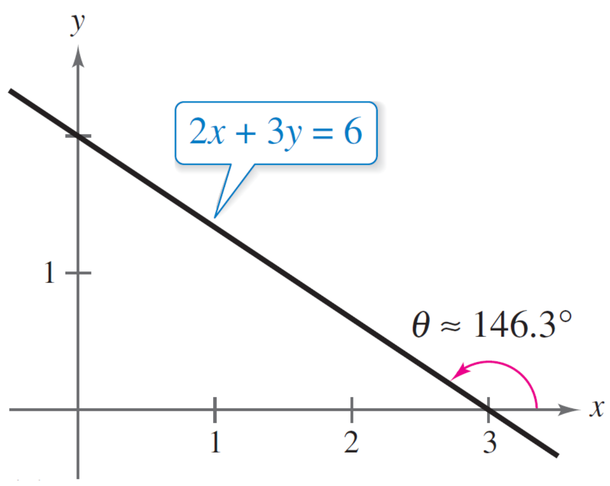


Figure 6.1(b)

The Angle Between Two Lines

The Angle Between Two Lines

When two distinct lines intersect and are nonperpendicular, their intersection forms two pairs of opposite angles.

One pair is acute and the other pair is obtuse. The smaller of these angles is the **angle between the two lines**.

If two lines have inclinations θ_1 and θ_2, where $\theta_1 < \theta_2$ and $\theta_2 - \theta_1 < \pi/2$, then the angle between the two lines is

$$\theta = \theta_2 - \theta_1.$$

The Angle Between Two Lines

As shown in Figure 6.2.

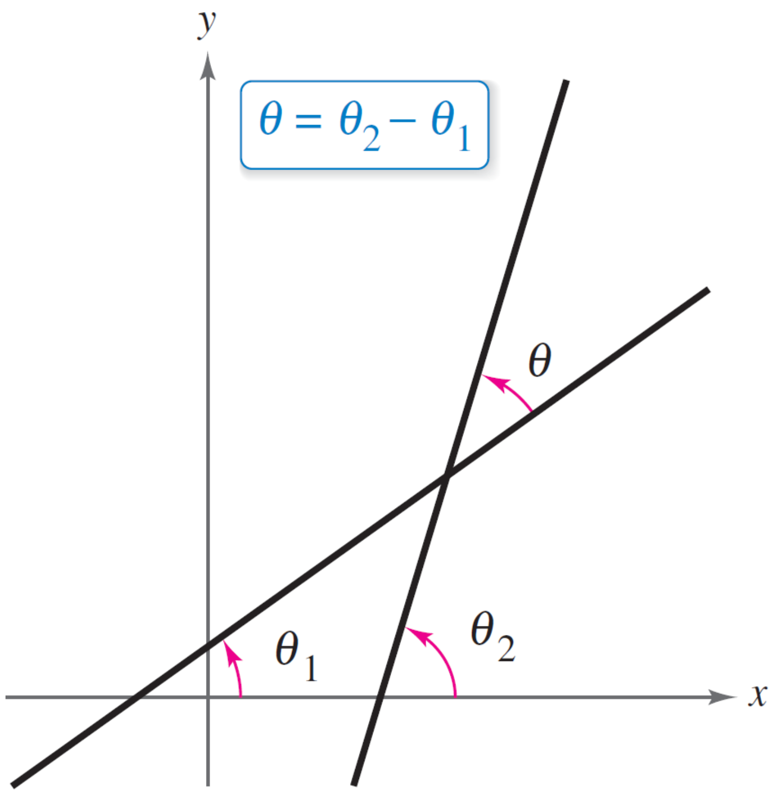


Figure 6.2

The Angle Between Two Lines

You can use the formula for the tangent of the difference of two angles

$$\tan\,\theta = \tan(\theta_2 - \theta_1)$$

$$= \frac{\tan\,\theta_2 - \tan\,\theta_1}{1 + \tan\,\theta_1 \tan\,\theta_2}$$

to obtain the formula for the angle between two lines.

The Angle Between Two Lines

Angle Between Two Lines

If two nonperpendicular lines have slopes m_1 and m_2, then the tangent of the angle between the two lines is

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|.$$

Example 2 – *Finding the Angle Between Two Lines*

Find the angle between $2x - y = 4$ and $3x + 4y = 12$.

Solution:

The two lines have slopes of $m_1 = 2$ and $m_2 = -\frac{3}{4}$, respectively. So, the tangent of the angle between the two lines is

$$\tan\theta = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right| = \left|\frac{(-3/4) - 2}{1 + (2)(-3/4)}\right| = \left|\frac{-11/4}{-2/4}\right| = \frac{11}{2}.$$

Example 2 – *Solution*

cont'd

Finally, you can conclude that the angle is

$$\theta = \arctan \frac{11}{2}$$

$$\approx 1.3909 \text{ radians}$$

$$\approx 79.7°$$

as shown in Figure 6.3.

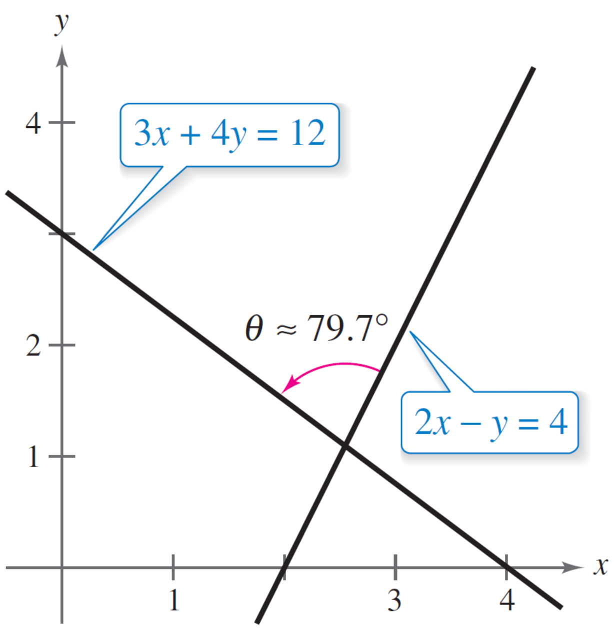


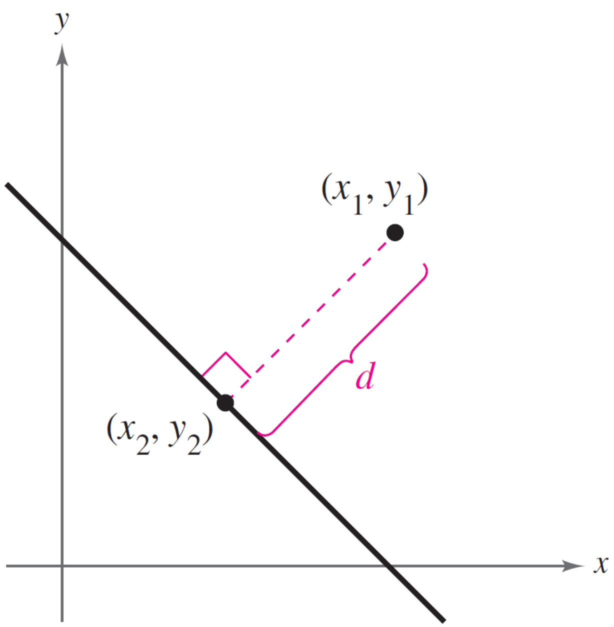
Figure 6.3

The Distance Between a Point and a Line

The Distance Between a Point and a Line

Finding the distance between a line and a point not on the line is an application of perpendicular lines.

This distance is defined as the length of the perpendicular line segment joining the point and the line, as shown below.



The Distance Between a Point and a Line

Distance Between a Point and a Line

The distance between the point (x_1, y_1) and the line $Ax + By + C = 0$ is

$$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}.$$

Remember that the values of *A*, *B*, and *C* in this distance formula correspond to the general equation of a line, $Ax + By + C = 0$.

Example 3 – *Finding the Distance Between a Point and a Line*

Find the distance between the point (4, 1) and the line $y = 2x + 1$.

Solution:

The general form of the equation is $-2x + y - 1 = 0$.

So, the distance between the point and the line is

$$d = \frac{|-2(4) + 1(1) + (-1)|}{\sqrt{(-2)^2 + 1^2}}$$

$$= \frac{8}{\sqrt{5}}$$

$$\approx 3.58 \text{ units.}$$

Example 3 – *Solution*

cont'd

The line and the point are shown in Figure 6.4.

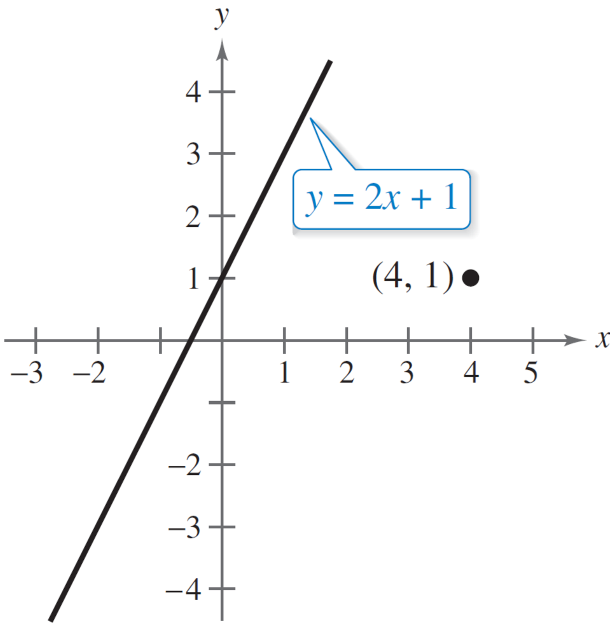


Figure 6.4