2 Analytic Trigonometry

2.4 Sum and Difference Formulas

Objective

- Use sum and difference formulas to evaluate trigonometric functions, verify identities, and solve trigonometric equations.

Using Sum and Difference Formulas

Using Sum and Difference Formulas

In this and the following section, you will study the uses of several trigonometric identities and formulas.

Sum and Difference Formulas

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u + v) = \frac{\tan u + \tan v}{1 - \tan u \tan v} \qquad \tan(u - v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Example 1 shows how **sum and difference formulas** can enable you to find exact values of trigonometric functions involving sums or differences of special angles.

Example 1 – *Evaluating a Trigonometric Function*

Find the exact value of $\sin \frac{\pi}{12}$.

Solution:

To find the *exact* value of sin π/12, use the fact that

$$\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}.$$

Consequently, the formula for sin(*u* – *v*) yields

$$\sin \frac{\pi}{12} = \sin\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$$

Example 1 – *Solution*

cont'd

$$= \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$$

$$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}.$$

Try checking this result on your calculator. You will find that $\sin \pi/12 \approx 0.259$.

Example 5 – *Proving a Cofunction identity*

Use a difference formula to prove the cofunction identity

$\cos\left(\frac{\pi}{2} - x\right)$ = sin *x*.

Solution:

Using the formula for cos(*u* – *v*), you have

$$\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$$

$$= (0)(\cos x) + (1)(\sin x)$$

$$= \sin x.$$

Using Sum and Difference Formulas

Sum and difference formulas can be used to rewrite expressions such as

$\sin\left(\theta + \frac{n\pi}{2}\right)$ and $\cos\left(\theta + \frac{n\pi}{2}\right)$, where *n* is an integer

as expressions involving only sin θ or cos θ.

The resulting formulas are called **reduction formulas**.

Example 7 – *Solving a Trigonometric Equation*

Find all solutions of $\sin[x + (\pi/4)] + \sin[x - (\pi/4)] = -1$ in the interval $[0, 2\pi)$.

Solution:

Algebraic Solution

Using sum and difference formulas, rewrite the equation as

$$\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4} + \sin x \cos \frac{\pi}{4} - \cos x \sin \frac{\pi}{4} = -1$$

$$2 \sin x \cos \frac{\pi}{4} = -1$$

Example 7 – *Solution*

cont'd

$$2(\sin x)\left(\frac{\sqrt{2}}{2}\right) = -1$$

$$\sin x = -\frac{1}{\sqrt{2}}$$

$$\sin x = -\frac{\sqrt{2}}{2}.$$

So, the only solutions in the interval $[0, 2\pi)$ are $x = 5\pi/4$ and $x = 7\pi/4$.