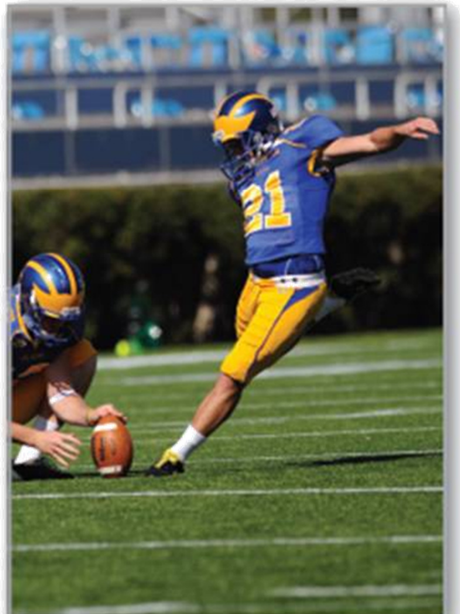
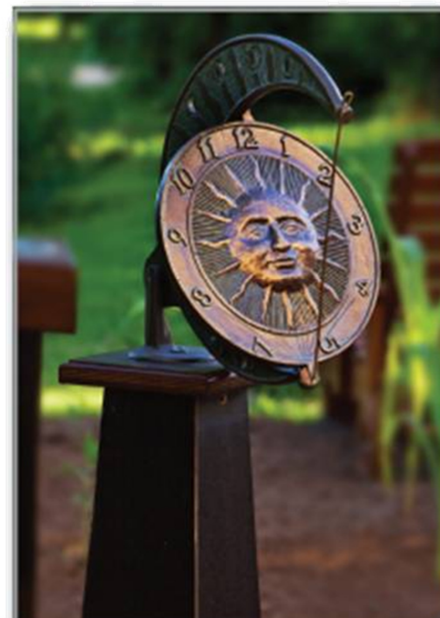
2 Analytic Trigonometry





2.1 Using Fundamental Identities

Objectives

- Recognize and write the fundamental trigonometric identities.
- Use the fundamental trigonometric identities to evaluate trigonometric functions, simplify trigonometric expressions, and rewrite trigonometric expressions.

Introduction

Introduction

You will learn how to use the fundamental identities to do the following.

1. Evaluate trigonometric functions.

2. Simplify trigonometric expressions.

3. Develop additional trigonometric identities.

4. Solve trigonometric equations.

Introduction

Fundamental Trigonometric Identities

Reciprocal Identities

$$\sin u = \frac{1}{\csc u} \qquad \cos u = \frac{1}{\sec u} \qquad \tan u = \frac{1}{\cot u}$$

$$\csc u = \frac{1}{\sin u} \qquad \sec u = \frac{1}{\cos u} \qquad \cot u = \frac{1}{\tan u}$$

Quotient Identities

$$\tan u = \frac{\sin u}{\cos u} \qquad \cot u = \frac{\cos u}{\sin u}$$

Pythagorean Identities

$$\sin^2 u + \cos^2 u = 1 \qquad 1 + \tan^2 u = \sec^2 u \qquad 1 + \cot^2 u = \csc^2 u$$

Introduction

cont'd

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - u\right) = \cos u \qquad \cos\left(\frac{\pi}{2} - u\right) = \sin u$$

$$\tan\left(\frac{\pi}{2} - u\right) = \cot u \qquad \cot\left(\frac{\pi}{2} - u\right) = \tan u$$

$$\sec\left(\frac{\pi}{2} - u\right) = \csc u \qquad \csc\left(\frac{\pi}{2} - u\right) = \sec u$$

Even/Odd Identities

$$\sin(-u) = -\sin u \qquad \cos(-u) = \cos u \qquad \tan(-u) = -\tan u$$

$$\csc(-u) = -\csc u \qquad \sec(-u) = \sec u \qquad \cot(-u) = -\cot u$$

Introduction

Pythagorean identities are sometimes used in radical form such as

$$\sin u = \pm\sqrt{1 - \cos^2 u}$$

or

$$\tan u = \pm\sqrt{\sec^2 u - 1}$$

where the sign depends on the choice of *u*.

Using the Fundamental Identities

Using the Fundamental Identities

One common application of trigonometric identities is to use given values of trigonometric functions to evaluate other trigonometric functions.

Example 1 – *Using Identities to Evaluate a Function*

Use the values $\sec u = -\frac{3}{2}$ and tan $u > 0$ to find the values of all six trigonometric functions.

Solution:

Using a reciprocal identity, you have

$$\cos u = \frac{1}{\sec u} = \frac{1}{-3/2} = -\frac{2}{3}.$$

Using a Pythagorean identity, you have

$$\sin^2 u = 1 - \cos^2 u \qquad \text{Pythagorean identity}$$

Example 1 – *Solution*

cont'd

$$= 1 - \left(-\frac{2}{3}\right)^2$$ Substitute $-\frac{2}{3}$ for cos *u*.

$$= \frac{5}{9}.$$ Simplify.

Because sec $u < 0$ and tan $u > 0$, it follows that u lies in Quadrant III.

Moreover, because sin u is negative when u is in Quadrant III, choose the negative root and obtain

$$\sin u = -\sqrt{5}/3.$$

Example 1 – *Solution*

cont'd

Knowing the values of the sine and cosine enables you to find the values of all six trigonometric functions.

$$\sin u = -\frac{\sqrt{5}}{3}$$

$$\cos u = -\frac{2}{3}$$

$$\tan u = \frac{\sin u}{\cos u} = \frac{-\sqrt{5}/3}{-2/3} = \frac{\sqrt{5}}{2}$$

Example 1 – *Solution*

cont'd

$$\csc u = \frac{1}{\sin u} = -\frac{3}{\sqrt{5}} = -\frac{3\sqrt{5}}{5}$$

$$\sec u = \frac{1}{\cos u} = -\frac{3}{2}$$

$$\cot u = \frac{1}{\tan u} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

Example 2 – *Simplifying a Trigonometric Expression*

Simplify

$$\sin x \cos^2 x - \sin x.$$

Solution:

First factor out a common monomial factor and then use a fundamental identity.

$\sin x \cos^2 x - \sin x = \sin x(\cos^2 x - 1)$ — Factor out common monomial factor.

$= -\sin x(1 - \cos^2 x)$ — Factor out -1.

$= -\sin x(\sin^2 x)$ — Pythagorean identity

$= -\sin^3 x$ — Multiply.

Using the Fundamental Identities

When factoring trigonometric expressions, it is helpful to find a special polynomial factoring form that fits the expression.

On occasion, factoring or simplifying can best be done by first rewriting the expression in terms of just *one* trigonometric function or in terms of *sine and cosine only*.

Example 7 – *Rewriting a Trigonometric Expression*

Rewrite $\dfrac{1}{1 + \sin x}$ so that it is *not* in fractional form.

Solution:

From the Pythagorean identity
$\cos^2 x = 1 - \sin^2 x = (1 - \sin x)(1 + \sin x)$
multiplying both the numerator and the denominator by $(1 - \sin x)$ will produce a monomial denominator.

$$\frac{1}{1 + \sin x} = \frac{1}{1 + \sin x} \cdot \frac{1 - \sin x}{1 - \sin x}$$

Multiply numerator and denominator by $(1 - \sin x)$.

$$= \frac{1 - \sin x}{1 - \sin^2 x}$$

Multiply.

Example 7 – *Solution*

cont'd

$$= \frac{1 - \sin x}{\cos^2 x}$$

Pythagorean identity

$$= \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}$$

Write as separate fractions.

$$= \frac{1}{\cos^2 x} - \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$$

Product of fractions

$$= \sec^2 x - \tan x \sec x$$

Reciprocal and quotient identities