1 Trigonometry

1.8 Applications and Models

Objectives

- Solve real-life problems involving right triangles.
- Solve real-life problems involving directional bearings.
- Solve real-life problems involving harmonic motion.

Applications Involving Right Triangles

Applications Involving Right Triangles

In this section, the three angles of a right triangle are denoted by the letters A, B, and C (where C is the right angle), and the lengths of the sides opposite these angles by the letters a, b, and c, respectively (where c is the hypotenuse).

Example 2 – *Finding a Side of a Right Triangle*

A safety regulation states that the maximum angle of elevation for a rescue ladder is 72°. A fire department's longest ladder is 110 feet. What is the maximum safe rescue height?

Solution:

A sketch is shown in Figure 1.57.

From the equation $\sin A = a/c$, it follows that

$$a = c \sin A$$

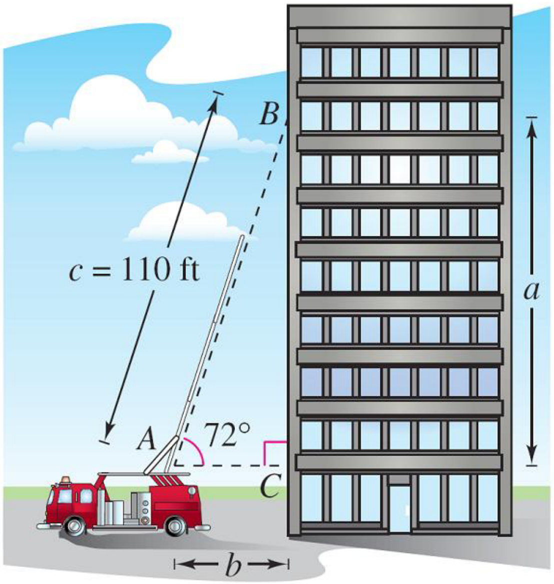


Figure 1.57

Example 2 – *Solution*

cont'd

$$= 110 \sin 72^\circ$$

$$\approx 104.6.$$

So, the maximum safe rescue height is about 104.6 feet above the height of the fire truck.

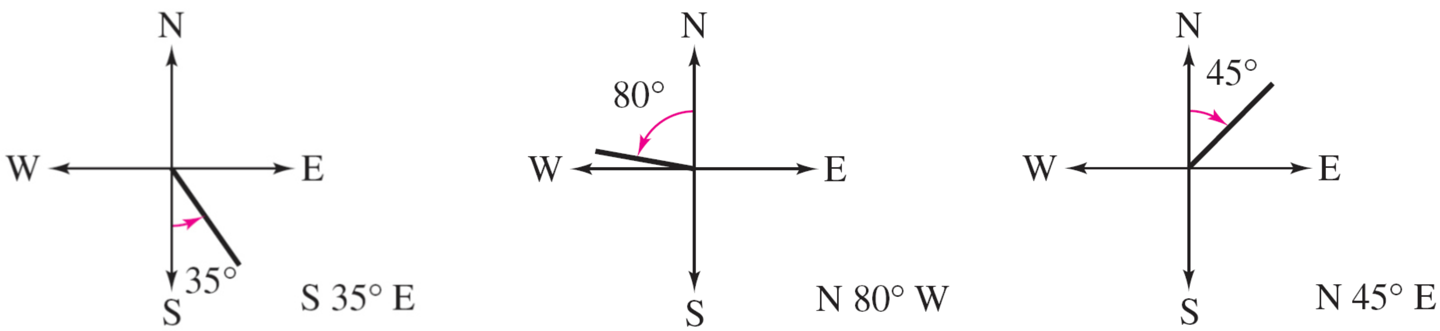
Trigonometry and Bearings

Trigonometry and Bearings

In surveying and navigation, directions can be given in terms of **bearings.**

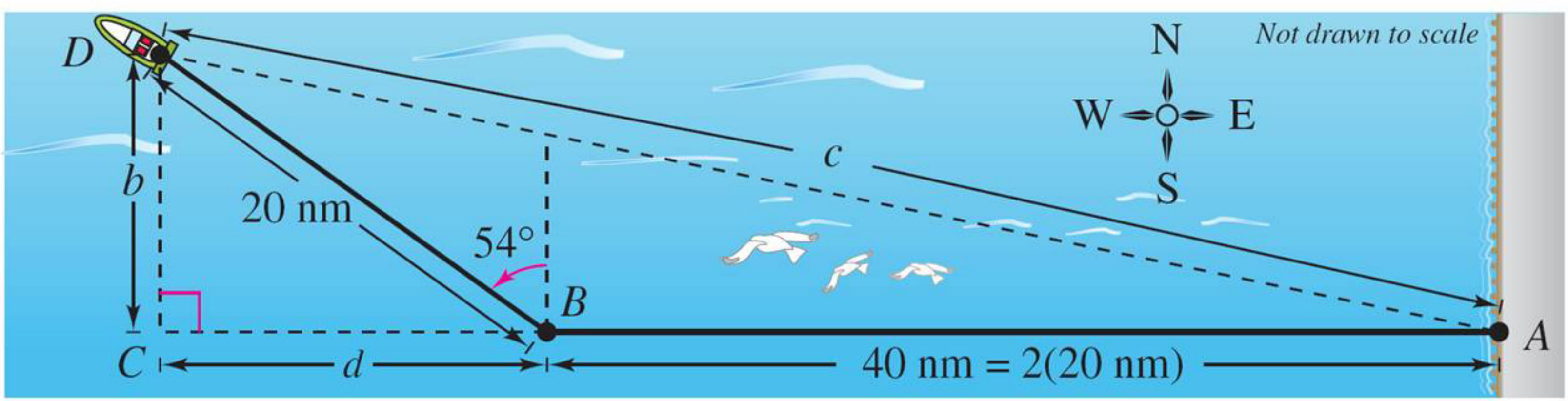
A bearing measures the acute angle that a path or line of sight makes with a fixed north-south line.

For instance, the bearing S 35° E, shown below, means 35 degrees east of south.



Example 5 – *Finding Directions in Terms of Bearings*

A ship leaves port at noon and heads due west at 20 knots, or 20 nautical miles (nm) per hour. At 2 P.M. the ship changes course to N 54° W, as shown below. Find the ship's bearing and distance from the port of departure at 3 P.M.



Example 5 – *Solution*

For triangle *BCD*, you have

$$B = 90° - 54° = 36°.$$

The two sides of this triangle can be determined to be

$$b = 20 \sin 36° \qquad \text{and} \qquad d = 20 \cos 36°.$$

For triangle *ACD*, you can find angle *A* as follows.

$$\tan A = \frac{b}{d + 40} = \frac{20 \sin 36°}{20 \cos 36° + 40} \approx 0.2092494$$

$$A \approx \arctan 0.2092494 \approx 0.2062732 \text{ radian} \approx 11.82°$$

Example 5 – *Solution*

cont'd

The angle with the north-south line is 90° – 11.82°= 78.18°.

So, the bearing of the ship is N 78.18° W.

Finally, from triangle *ACD*, you have sin *A* = *b*/ *c*, which yields

$$c = \frac{b}{\sin A} = \frac{20 \sin 36^\circ}{\sin 11.82^\circ}$$

$\approx$ 57.4 nautical miles. Distance from port

Harmonic Motion

Harmonic Motion

The periodic nature of the trigonometric functions is useful for describing the motion of a point on an object that vibrates, oscillates, rotates, or is moved by wave motion.

Harmonic Motion

For example, consider a ball that is bobbing up and down on the end of a spring, as shown in Figure 1.60.

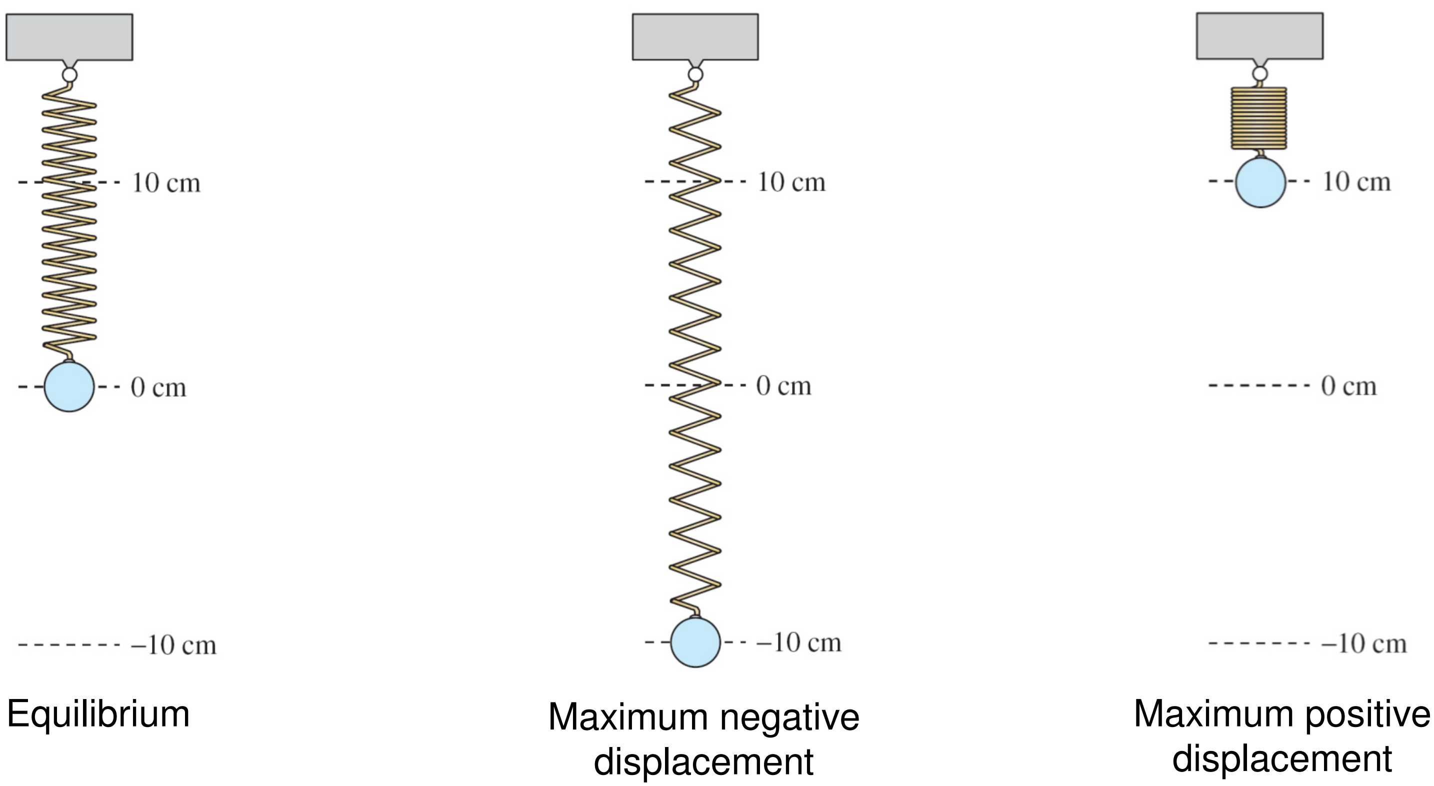


Figure 1.60

Harmonic Motion

Suppose that 10 centimeters is the maximum distance the ball moves vertically upward or downward from its equilibrium (at rest) position.

Suppose further that the time it takes for the ball to move from its maximum displacement above zero to its maximum displacement below zero and back again is $t = 4$ seconds.

Assuming the ideal conditions of perfect elasticity and no friction or air resistance, the ball would continue to move up and down in a uniform and regular manner.

Harmonic Motion

From this spring you can conclude that the period (time for one complete cycle) of the motion is

Period = 4 seconds

its amplitude (maximum displacement from equilibrium) is

Amplitude = 10 centimeters

and its **frequency** (number of cycles per second) is

Frequency = $\frac{1}{4}$ cycle per second.

Motion of this nature can be described by a sine or cosine function and is called **simple harmonic motion.**

Harmonic Motion

Definition of Simple Harmonic Motion

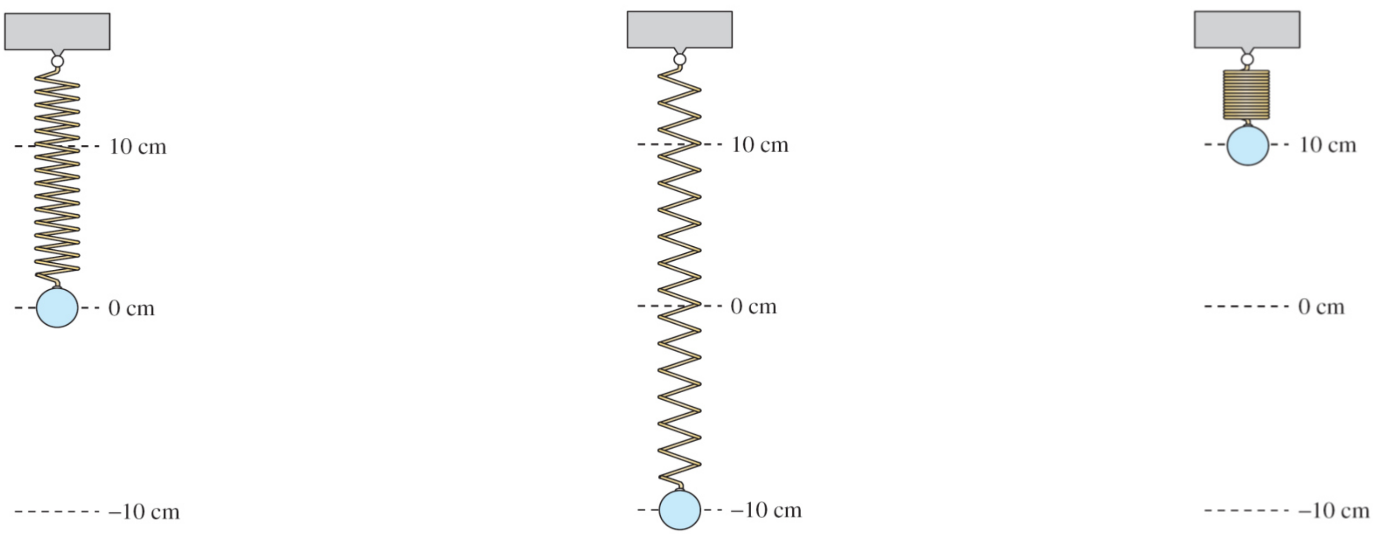
A point that moves on a coordinate line is in **simple harmonic motion** when its distance d from the origin at time t is given by either

$$d = a \sin \omega t \quad \text{or} \quad d = a \cos \omega t$$

where a and ω are real numbers such that $\omega > 0$. The motion has amplitude $|a|$, period $\dfrac{2\pi}{\omega}$, and frequency $\dfrac{\omega}{2\pi}$.

Example 6 – *Simple Harmonic Motion*

Write an equation for the simple harmonic motion of the ball described in Figure 1.60, where the period is 4 seconds. What is the frequency of this harmonic motion?



Equilibrium

Maximum negative displacement

Maximum positive displacement

Figure 1.60

Example 6 – *Solution*

Because the spring is at equilibrium ($d = 0$) when $t = 0$, use the equation

$$d = a \sin \omega t.$$

Moreover, because the maximum displacement from zero is 10 and the period is 4, you have the following.

$$\text{Amplitude} = |a| = 10$$

$$\text{Period} = \frac{2\pi}{\omega} = 4 \quad \Rightarrow \quad \omega = \frac{\pi}{2}$$

Example 6 – *Solution*

cont'd

Consequently, an equation of motion is

$$d = 10 \sin \frac{\pi}{2} t.$$

Note that the choice of $a = 10$ or $a = -10$ depends on whether the ball initially moves up or down.

The frequency is

$$\text{Frequency} = \frac{\omega}{2\pi}$$

$$= \frac{\pi/2}{2\pi}$$

$$= \frac{1}{4} \text{ cycle per second.}$$

Harmonic Motion

One illustration of the relationship between sine waves and harmonic motion is in the wave motion that results when a stone is dropped into a calm pool of water.

The waves move outward in roughly the shape of sine (or cosine) waves, as shown in Figure 1.61.

Figure 1.61

Harmonic Motion

As an example, suppose you are fishing and your fishing bobber is attached so that it does not move horizontally.

As the waves move outward from the dropped stone, your fishing bobber will move up and down in simple harmonic motion, as shown in Figure 1.62.

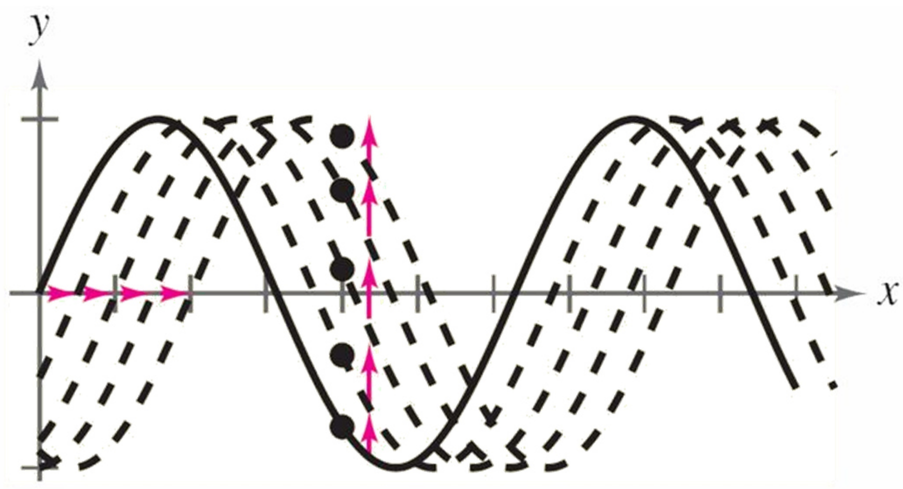


Figure 1.62