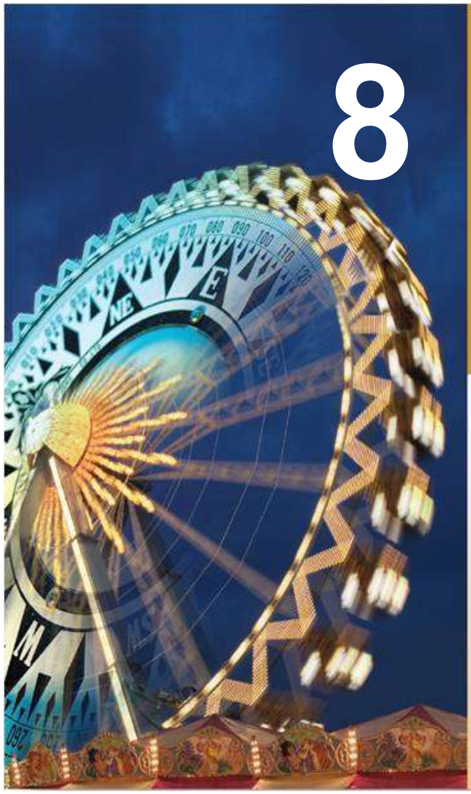

8 Complex Numbers and Polar Coordinates

SECTION 8.6

Equations in Polar Coordinates and Their Graphs

Learning Objectives

1. Graph an equation in polar coordinates by plotting points.
2. Graph an equation in polar coordinates by analyzing a graph in rectangular coordinates.
3. Graph an equation in polar coordinates using a graphing calculator.
4. Identify the graph of a polar equation.

Equations in Polar Coordinates and Their Graphs

In this section, we will consider the graphs of polar equations.

The solutions to these equations are ordered pairs (r, θ), where r and θ are the polar coordinates.

Example 1

Sketch the graph of $r = 6 \sin \theta$.

Solution:

We can find ordered pairs (r, θ) that satisfy the equation by making a table. Table 1 is a little different from the ones we made for rectangular coordinates.

θ	$r = 6 \sin \theta$	r	(r, θ)
$0°$	$r = 6 \sin 0° = 0$	0	$(0, 0°)$
$30°$	$r = 6 \sin 30° = 3$	3	$(3, 30°)$
$45°$	$r = 6 \sin 45° = 4.2$	4.2	$(4.2, 45°)$
$60°$	$r = 6 \sin 60° = 5.2$	5.2	$(5.2, 60°)$
$90°$	$r = 6 \sin 90° = 6$	6	$(6, 90°)$
$120°$	$r = 6 \sin 120° = 5.2$	5.2	$(5.2, 120°)$

Table 1

Example 1 – *Solution*

cont'd

θ	$r = 6 \sin \theta$	r	(r, θ)
$135°$	$r = 6 \sin 135° = 4.2$	4.2	$(4.2, 135°)$
$150°$	$r = 6 \sin 150° = 3$	3	$(3, 150°)$
$180°$	$r = 6 \sin 180° = 0$	0	$(0, 180°)$
$210°$	$r = 6 \sin 210° = -3$	-3	$(-3, 210°)$
$225°$	$r = 6 \sin 225° = -4.2$	-4.2	$(-4.2, 225°)$
$240°$	$r = 6 \sin 240° = -5.2$	-5.2	$(-5.2, 240°)$
$270°$	$r = 6 \sin 270° = -6$	-6	$(-6, 270°)$
$300°$	$r = 6 \sin 300° = -5.2$	-5.2	$(-5.2, 300°)$
$315°$	$r = 6 \sin 315° = -4.2$	-4.2	$(-4.2, 315°)$
$330°$	$r = 6 \sin 330° = -3$	-3	$(-3, 330°)$
$360°$	$r = 6 \sin 360° = 0$	0	$(0, 360°)$

Table 1(continued)

Example 1 – *Solution*

cont'd

With polar coordinates, we substitute convenient values for θ and then use the equation to find corresponding values of r. Let's use multiples of 30° and 45° for θ.

Plotting each point on a polar coordinate system and then drawing a smooth curve through them, we have the graph in Figure 2.

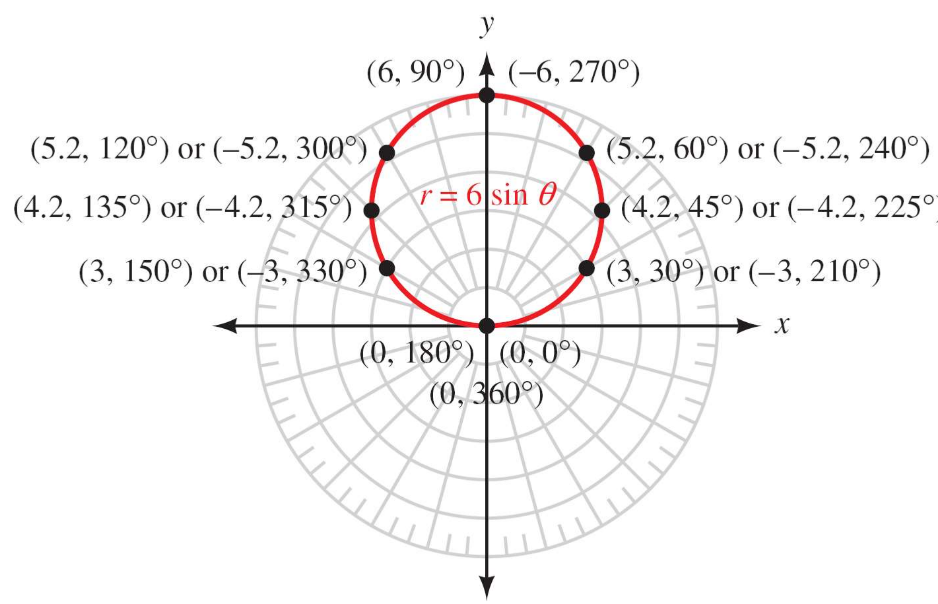

Figure 2

Equations in Polar Coordinates and Their Graphs

Could we have found the graph of $r = 6 \sin \theta$ in Example 1 without making a table? The answer is yes, there are a couple of other ways to do so.

One way is to convert to rectangular coordinates and see if we recognize the graph from the rectangular equation.

We begin by replacing $\sin \theta$ with y/r.

$$r = 6 \sin \theta$$

$$r = 6 \frac{y}{r} \qquad \sin \theta = \frac{y}{r}$$

Equations in Polar Coordinates and Their Graphs

$$r^2 = 6y \qquad \text{Multiply both sides by } r$$

$$x^2 + y^2 = 6y \qquad r^2 = x^2 + y^2$$

The equation is now written in terms of rectangular coordinates.

If we add –6*y* to both sides and then complete the square on *y*, we will obtain the rectangular equation of a circle with center at (0, 3) and a radius of 3.

$$x^2 + y^2 - 6y = 0 \qquad \text{Add } -6y \text{ to both sides}$$

$$x^2 + y^2 - 6y + 9 = 9 \qquad \text{Complete the square on } y \text{ by adding 9 to both sides}$$

$$x^2 + (y - 3)^2 = 3^2 \qquad \text{Standard form for the equation of a circle}$$

Equations in Polar Coordinates and Their Graphs

This method of graphing, by changing to rectangular coordinates, works well only in some cases.

In Example 2, we will look at another method of graphing polar equations that does not depend on the use of a table.

Example 2

Sketch the graph of $r = 4 \sin 2\theta$.

Solution:

One way to visualize the relationship between r and θ as given by the equation $r = 4 \sin 2\theta$ is to sketch the graph of $y = 4 \sin 2x$ on a rectangular coordinate system.

(We have been using degree measure for our angles in polar coordinates, so we will label the x-axis for the graph of $y = 4 \sin 2x$ in degrees rather than radians as we usually do.)

Example 2 – *Solution*

cont'd

The graph of $y = 4 \sin 2x$ will have an amplitude of 4 and a period of 360°/2 = 180°.

Figure 6 shows the graph of $y = 4 \sin 2x$ between 0° and 360°.

As you can see in Figure 6, as x goes from 0° to 45°, y goes from 0 to 4.

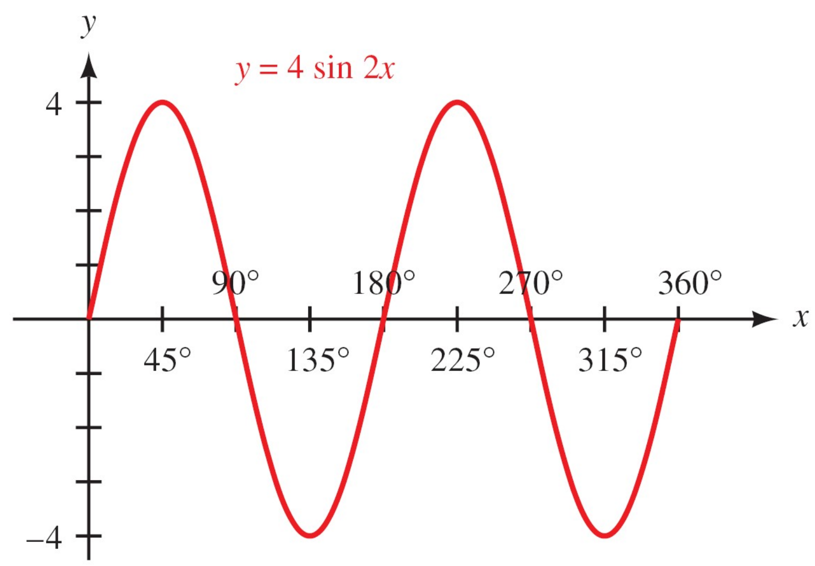

Figure 6

Example 2 – *Solution*

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This means that, for the equation $r = 4 \sin 2\theta$, as θ goes from 0° to 45°, r will go from 0 out to 4.

A diagram of this is shown in Figure 7.

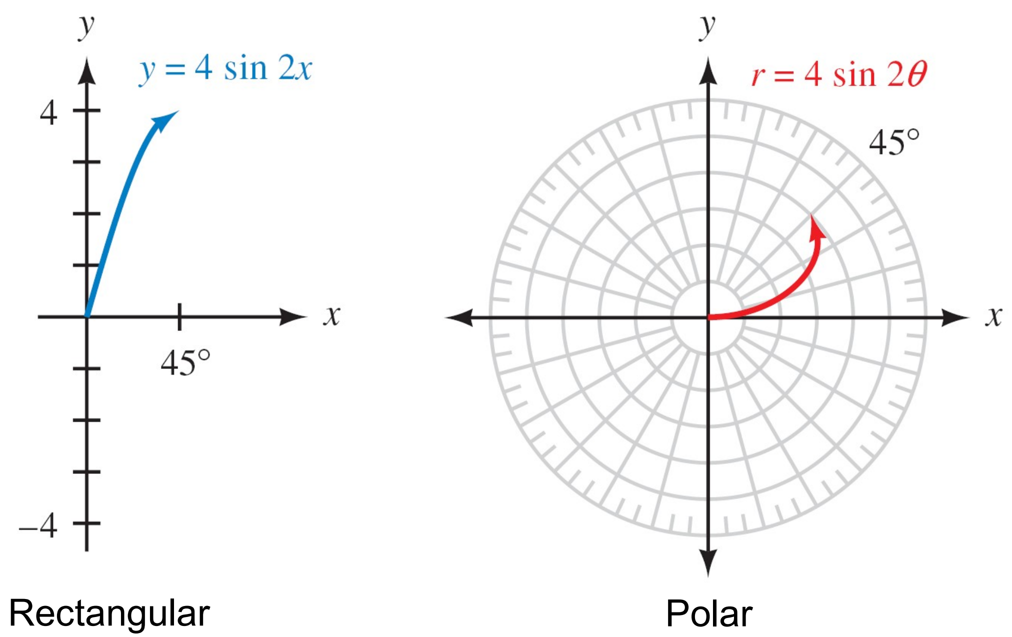


Figure 7

Example 2 – *Solution*

cont'd

As x continues from 45° to 90°, y decreases from 4 down to 0. Likewise, as θ rotates through 45° to 90°, r will decrease from 4 down to 0.

A diagram of this is shown in Figure 8. The numbers 1 and 2 in Figure 8 indicate the order in which those sections of the graph are drawn.

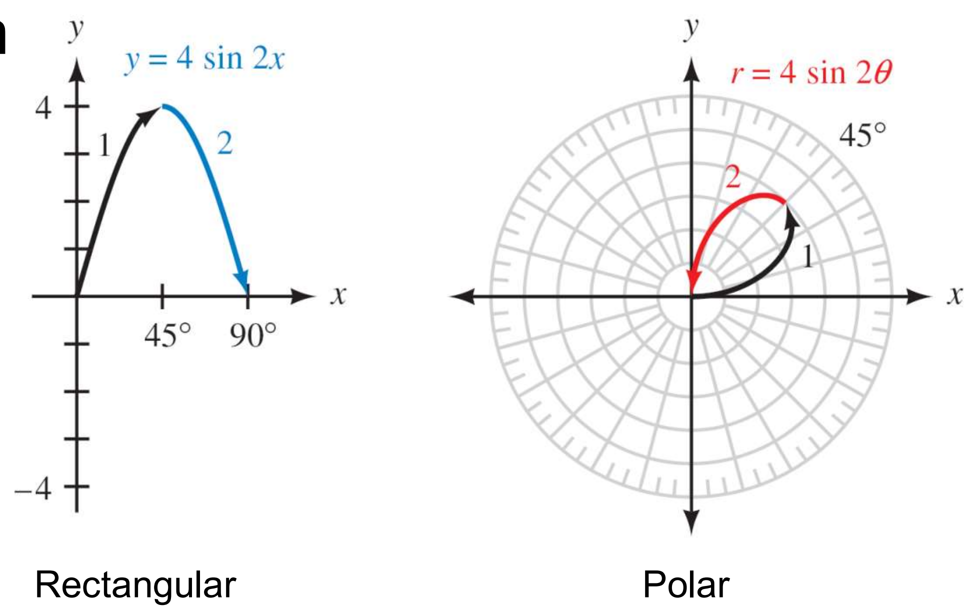


Figure 8

Example 2 – *Solution*

cont'd

If we continue to reason in this manner, we will obtain a good sketch of the graph of $r = 4 \sin 2\theta$ by watching how y is affected by changes in x on the graph of $y = 4 \sin 2x$.

Table 2 summarizes this information.

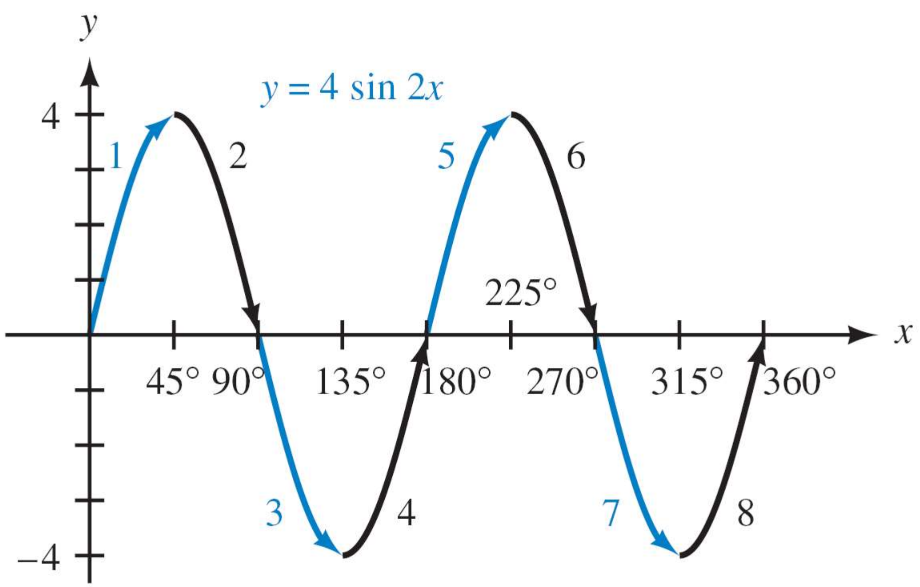
Reference Number on Graphs	Variations in x (or θ)	Corresponding Variations in y (or r)
1	0° to 45°	0 to 4
2	45° to 90°	4 to 0
3	90° to 135°	0 to −4
4	135° to 180°	−4 to 0
5	180° to 225°	0 to 4
6	225° to 270°	4 to 0
7	270° to 315°	0 to −4
8	315° to 360°	−4 to 0

Table 2

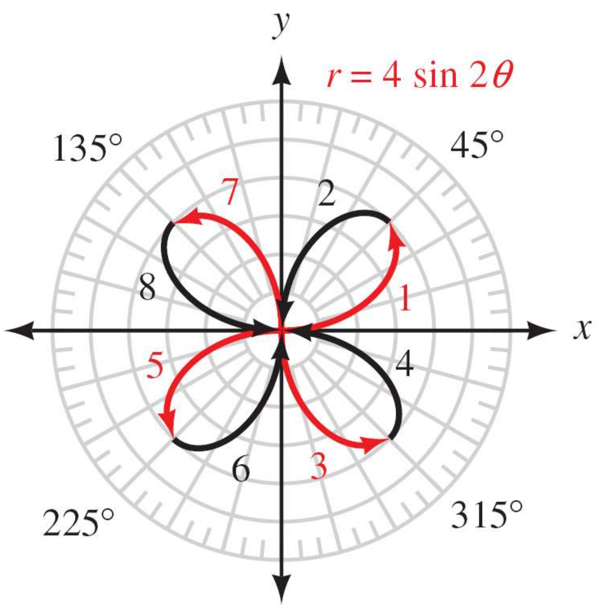
Example 2 – *Solution*

cont'd

Figure 9 contains the graphs of both $y = 4 \sin 2x$ and $r = 4 \sin 2\theta$.



Rectangular



Polar

Figure 9