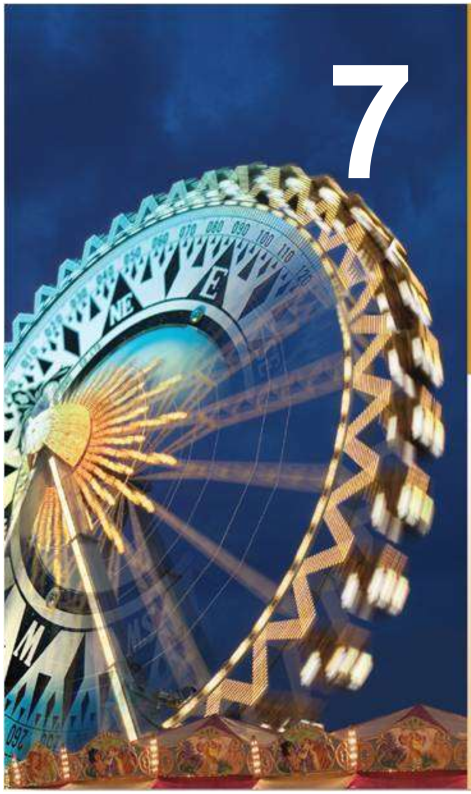

# 7 Triangles

SECTION 7.5

# Vectors: An Algebraic Approach

# Learning Objectives

1. Draw a vector in standard position.
2. Express a vector in terms of unit vectors **i** and **j**.
3. Find the magnitude of a vector.
4. Find a sum, difference, or scalar multiple of vectors.

# Standard Position

# Standard Position

We have known that a vector is in *standard position* when it is placed on a coordinate system so that its tail is located at the origin.

If the tip of the vector corresponds to the point (*a*, *b*), then the coordinates of this point provide a unique representation for the vector.

That is, the point (*a*, *b*) determines both the length of the vector and its direction, as shown in Figure 1.

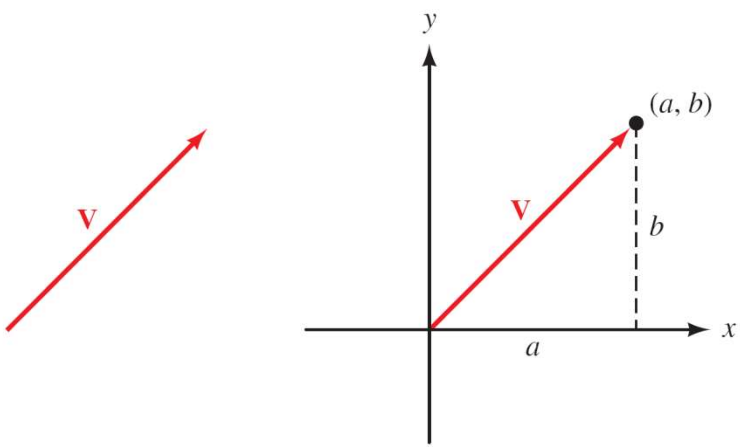


Figure 1

# Standard Position

To avoid confusion between the point ($a$, $b$) and the vector, which is the ray extending from the origin to the point, we will denote the vector as $\mathbf{v} = \langle a, b \rangle$.

We refer to this notation as *component form*. The $x$-coordinate, $a$, is called the *horizontal component* of **V**, and the $y$-coordinate, $b$, is called the *vertical component* of **V**.

# Magnitude

# Magnitude

As we have known, the magnitude of a vector is its length.

Referring to Figure 1, we can find the magnitude of the vector $\mathbf{v} = \langle a, b \rangle$ using the Pythagorean Theorem:

$$|\mathbf{v}| = \sqrt{a^2 + b^2}$$

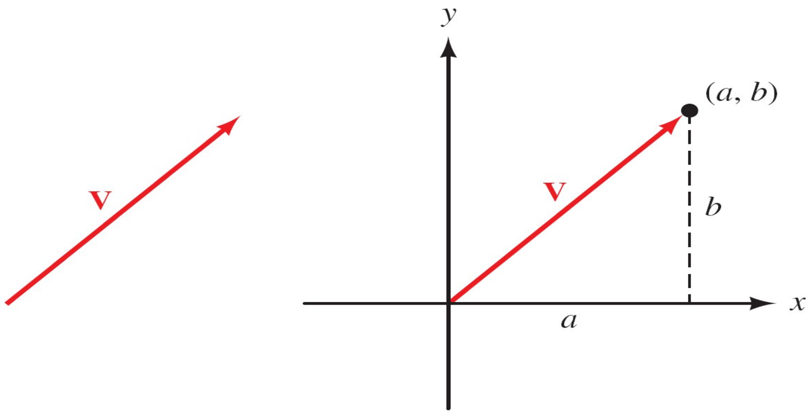


Figure 1

# Example 1

Draw the vector $\mathbf{v} = \langle 3, -4 \rangle$ in standard position and find its magnitude.

Solution:

We draw the vector by sketching an arrow from the origin to the point (3, –4), as shown in Figure 2.

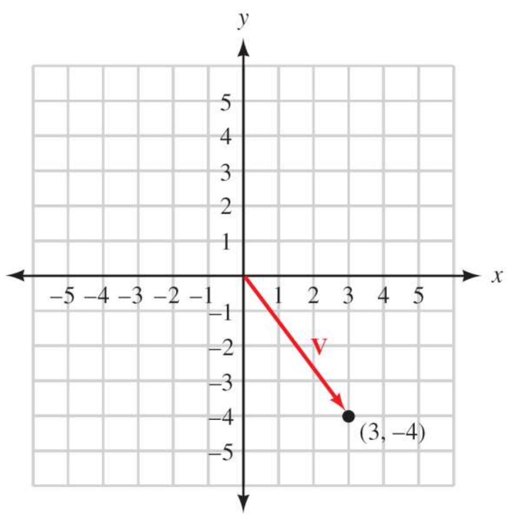


**Figure 2**

# Example 1 – *Solution*

cont'd

To find the magnitude of **V,** we find the positive square root of the sum of the squares of the horizontal and vertical components.

$$
\begin{aligned}
|\mathbf{V}| &= \sqrt{3^2 + (-4)^2} \\
&= \sqrt{9 + 16} \\
&= \sqrt{25} \\
&= 5
\end{aligned}
$$

# Addition and Subtraction with Algebraic Vectors

# Addition and Subtraction with Algebraic Vectors

Adding and subtracting vectors written in component form is simply a matter of adding (or subtracting) the horizontal components and adding (or subtracting) the vertical components.

Figure 3 shows the vector sum of vectors
$\mathbf{U} = \langle 6, 2 \rangle$ and $\mathbf{V} = \langle -3, 5 \rangle$.

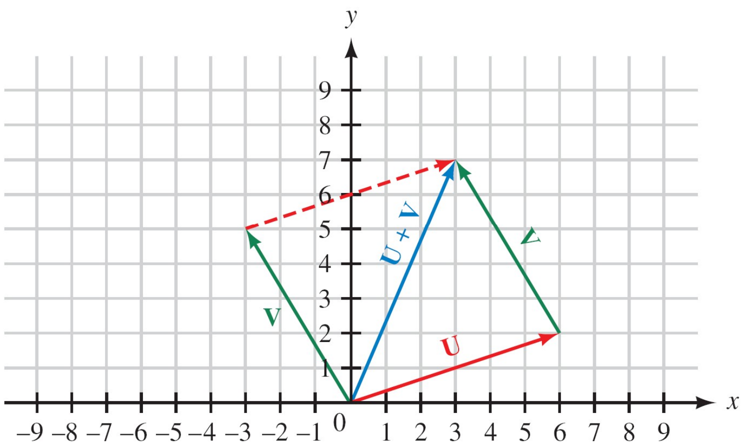


**Figure 3**

# Addition and Subtraction with Algebraic Vectors

Figure 4 shows the difference of vectors **U** and **V**. As the diagram in Figure 4 indicates, subtraction of algebraic vectors can be accomplished by subtracting corresponding components.

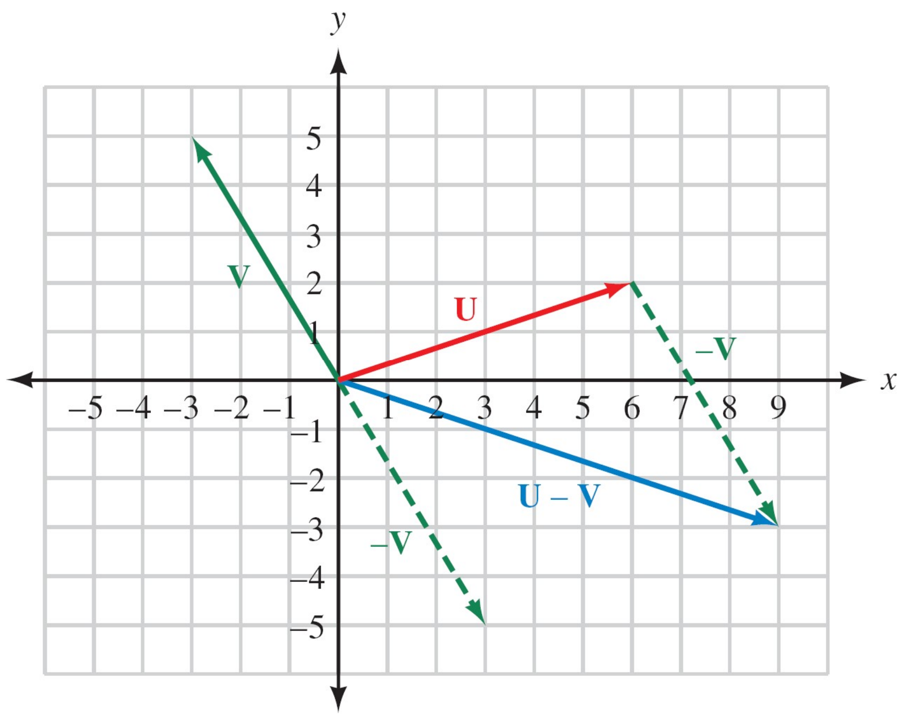


**Figure 4**

# Scalar Multiplication

# Scalar Multiplication

To multiply a vector in component form by a scalar (real number) we multiply each component of the vector by the scalar.

Figure 5 shows the vector $\mathbf{V} = \langle 2, 3 \rangle$ and the vector 3**V**.

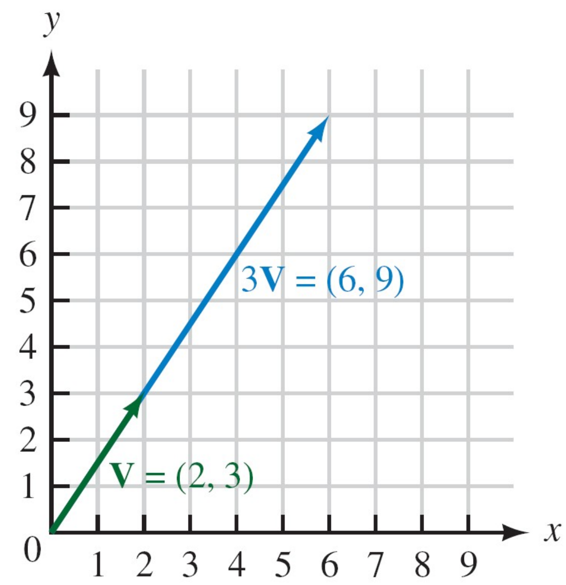


**Figure 5**

# Example 2

If $\mathbf{U} = \langle 5, -3 \rangle$ and $\mathbf{V} = \langle -6, 4 \rangle$, find

**a.** $\mathbf{U} + \mathbf{V}$

**b.** $4\mathbf{U} - 5\mathbf{V}$

Solution:

**a.** $\mathbf{U} + \mathbf{V} = \langle 5, -3 \rangle + \langle -6, 4 \rangle$

$$= \langle 5 - 6, -3 + 4 \rangle$$

$$= \langle -1, 1 \rangle$$

**b.** $4\mathbf{U} - 5\mathbf{V} = 4\langle 5, -3 \rangle - 5\langle -6, 4 \rangle$

$$= \langle 20, -12 \rangle + \langle 30, -20 \rangle$$

# Example 2 – *Solution*

cont'd

$$= \langle 20, -12 \rangle + \langle 30, -20 \rangle$$

$$= \langle 20 + 30, -12 - 20 \rangle$$

$$= \langle 50, -32 \rangle$$

# Component Vector Form

# Component Vector Form

Another way to represent a vector algebraically is to express the vector as the sum of a horizontal vector and a vertical vector.

As we have known, any vector **V** can be written in terms of its horizontal and vertical component vectors, $\mathbf{V}_x$, and $\mathbf{V}_y$, respectively.

To do this, we need to define two special vectors.

# Component Vector Form

**DEFINITION**

The vector that extends from the origin to the point (1, 0) is called the *unit horizontal vector* and is denoted by $\mathbf{i}$. The vector that extends from the origin to the point (0, 1) is called the *unit vertical vector* and is denoted by $\mathbf{j}$. Figure 6 shows the vectors $\mathbf{i}$ and $\mathbf{j}$.

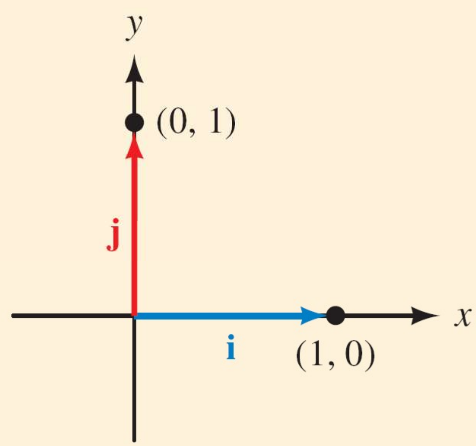


Figure 6

**Note:**

A *unit vector* is any vector whose magnitude is 1. Using our previous component notation, we would write $\mathbf{i} = \langle 1, 0 \rangle$ and $\mathbf{j} = \langle 0, 1 \rangle$.

# Example 3

Write the vector $\mathbf{v} = \langle 3, 4 \rangle$ in terms of the unit vectors **i** and **j**.

Solution:
From the origin, we must go three units in the positive *x*-direction, and then four units in the positive *y*-direction, to locate the terminal point of **V** at (3, 4).

Because **i** is a vector of length 1 in the positive *x*-direction, 3**i** will be a vector of length 3 in that same direction.

Likewise, 4**j** will be a vector of length 4 in the positive *y*-direction.

# Example 3 – *Solution*

cont'd

As shown in Figure 7, **V** is the sum of vectors 3**i** and 4**j**.

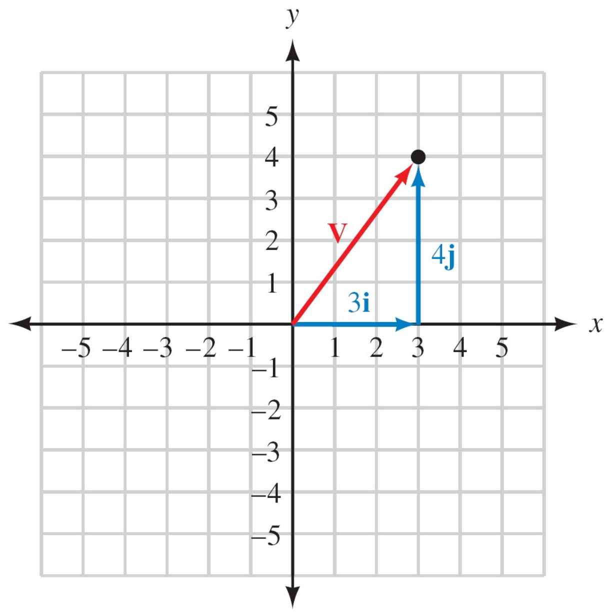


**Figure 7**

Therefore, we can write **V** in terms of the unit vectors **i** and **j** as

$$\mathbf{V} = 3\mathbf{i} + 4\mathbf{j}$$

# Component Vector Form

In Example 3, the vector 3**i** is the *horizontal vector component* of **V**, which we have previously referred to as $\mathbf{V}_x$.

Likewise, 4**j** is the *vertical vector component* of **V**, previously referred to as $\mathbf{V}_y$.

Notice that the coefficients of these vectors are simply the coordinates of the terminal point of **V**. That is,

$$\mathbf{V} = \langle 3, 4 \rangle = 3\mathbf{i} + 4\mathbf{j}$$

We refer to the notation **V** = 3**i** + 4**j** as *vector component form*.

# Component Vector Form

Every vector **V** can be written in terms of horizontal and vertical components and the unit vectors **i** and **j**.

Here is a summary of the information we have developed to this point.

**ALGEBRAIC VECTORS**

If **i** is the unit vector from (0, 0) to (1, 0), and **j** is the unit vector from (0, 0) to (0, 1), then any vector **V** can be written as

$$\mathbf{V} = a\mathbf{i} + b\mathbf{j} = \langle a, b \rangle$$

where $a$ and $b$ are real numbers (Figure 8).

The magnitude of **V** is

$$|\mathbf{V}| = \sqrt{a^2 + b^2}$$

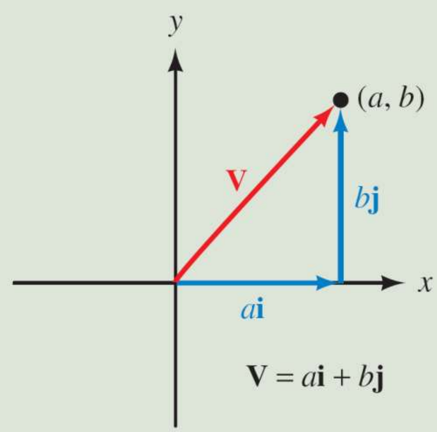


Figure 8