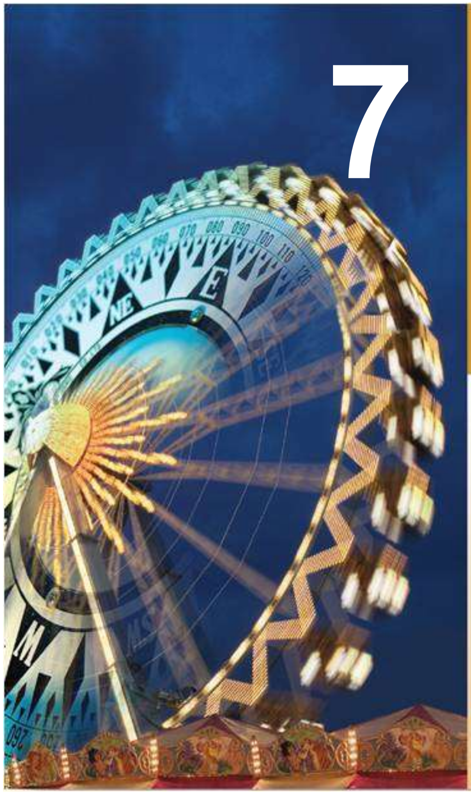

7 Triangles

SECTION 7.1

The Law of Sines

Learning Objectives

1. Use the law of sines to find a missing side in an oblique triangle.

2. Solve a real-life problem using the law of sines.

3. Use vectors and the law of sines to solve an applied problem.

The Law of Sines

We will now look at situations that involve *oblique triangles*, which are triangles that do not have a right angle.

Every triangle has three sides and three angles. To solve any triangle, we must know at least three of these six values. Refer Figure 1.

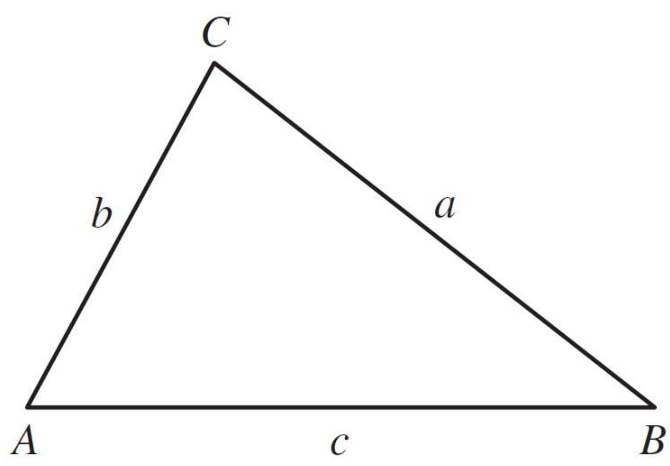

Figure 1

The Law of Sines

Table 1 summarizes the possible cases and shows which method can be used to solve the triangle in each case.

Case		Method
AAA	*Angle-angle-angle* This case cannot be solved because knowing all three angles does not determine a unique triangle. There are an infinite number of similar triangles that share the same angles.	None
AAS	*Angle-angle-side* Given two angles and a side opposite one of the angles, a unique triangle is determined.	Law of sines
ASA	*Angle-side-angle* Given two angles and the included side, a unique triangle is determined.	

Table 1

The Law of Sines

Case		Method
SAS	*Side-angle-side* Given two sides and the included angle, a unique triangle is determined.	Law of cosines
SSS	*Side-side-side* Given all three sides, a unique triangle is determined.	
SSA	*Side-side-angle* Given two sides and an angle opposite one of the sides, there may be one, two, or no triangles that are possible. This is known as the ambiguous case.	Law of sines or Law of cosines

Solving Oblique Triangles

Table 1(continued)

The Law of Sines

There are many relationships that exist between the sides and angles in a triangle.

One such relationship is called the *law of sines*, which states that the ratio of the sine of an angle to the length of the side opposite that angle is constant in any triangle.

LAW OF SINES

Given triangle ABC shown in Figure 2,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or, equivalently,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

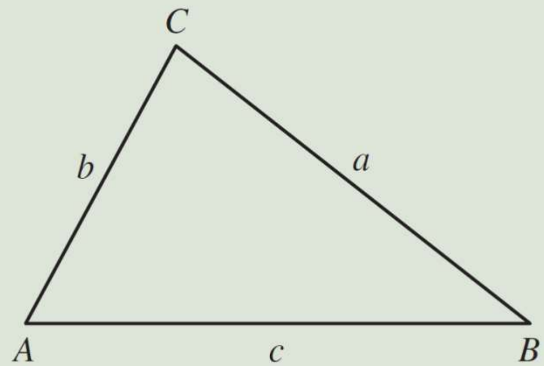

Figure 2

Two Angles and One Side

Example 1

In triangle *ABC*, *A* = 30°, *B* = 70°, and *a* = 8.0 cm. Find the length of side *c*.

Solution:

We begin by drawing a picture of triangle *ABC* (it does not have to be accurate) and labeling it with the information we have been given (Figure 5).

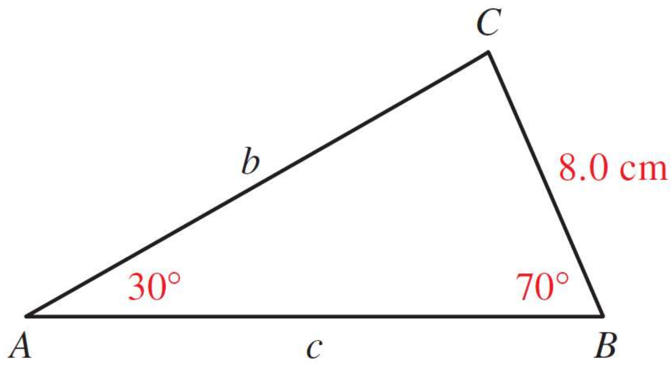

Figure 5

Example 1 – *Solution*

cont'd

When we use the law of sines, we must have one of the ratios given to us. In this case, since we are given *a* and *A*, we have the ratio $\frac{a}{\sin A}$.

To solve for *c*, we need to first find angle *C*. The sum of the angles in any triangle is 180°, so we have

$$
\begin{aligned}
C &= 180^\circ - (A + B) \\
&= 180^\circ - (30^\circ + 70^\circ) \\
&= 80^\circ
\end{aligned}
$$

Example 1 – *Solution*

cont'd

To find side *c*, we use the following two ratios given in the law of sines.

$$\frac{c}{\sin C} = \frac{a}{\sin A}$$

To solve for *c*, we multiply both sides by sin *C* and then substitute.

$$c = \frac{a \sin C}{\sin A} \qquad \text{Multiply both sides by } \sin C$$

Example 1 – *Solution*

cont'd

$$= \frac{8.0 \sin 80^\circ}{\sin 30^\circ}$$ Substitute in known values

$$= \frac{8.0(0.9848)}{0.5000}$$ Calculator

$$= 16 \text{ cm}$$ To two significant digits

Example 3

Figure 7 is a diagram of a shot put ring. The shot is tossed (put) from the left and lands at *A*.

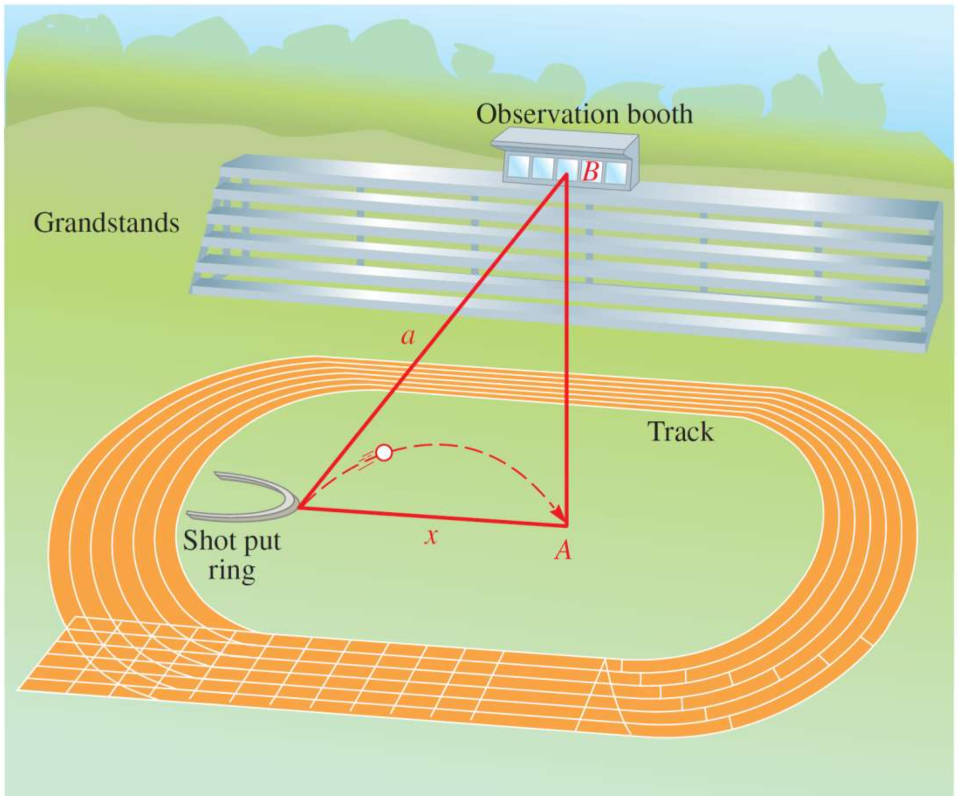

Figure 7

Example 3

cont'd

A small electronic device is then placed at *A* (there is usually a dent in the ground where the shot lands, so it is easy to find where to place the device).

The device at *A* sends a signal to a booth in the stands that gives the measures of angles *A* and *B*. The distance *a* is found ahead of time.

Find distance *x* in Figure 7 if $a = 562$ ft, $B = 5.7°$, and $A = 85.3°$.

Example 3 – *Solution*

To find the distance *x*, the law of sines is used.

$$\frac{x}{\sin B} = \frac{a}{\sin A} \Rightarrow x = \frac{a \sin B}{\sin A}$$

$$x = \frac{a \sin B}{\sin A} = \frac{562 \sin 5.7^\circ}{\sin 85.3^\circ}$$

$$= 56.0 \text{ ft}$$ To three significant digits

Example 6

A traffic light weighing 22 pounds is suspended by two wires as shown in Figure 10. Find the magnitude of the tension in wire *AB*, and the magnitude of the tension in wire *AC*.

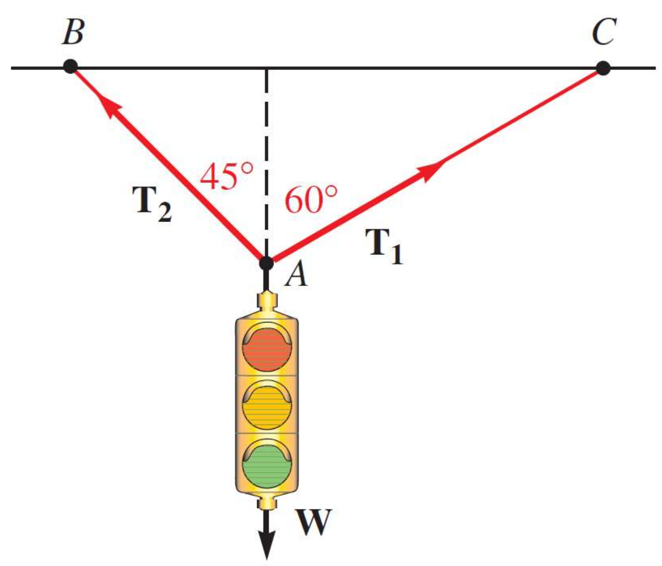

Figure 10

Example 6 – *Solution*

We assume that the traffic light is not moving and is therefore in a state of *static equilibrium*.

When an object is in this state, the sum of the forces acting on the object must be 0.

It is because of this fact that we can redraw the vectors from Figure 10 and be sure that they form a closed triangle.

Example 6 – *Solution*

cont'd

Figure 11 shows a convenient redrawing of the two tension vectors $\mathbf{T_1}$ and $\mathbf{T_2}$, and the vector $\mathbf{W}$ that is due to gravity. Notice that we have the case ASA.

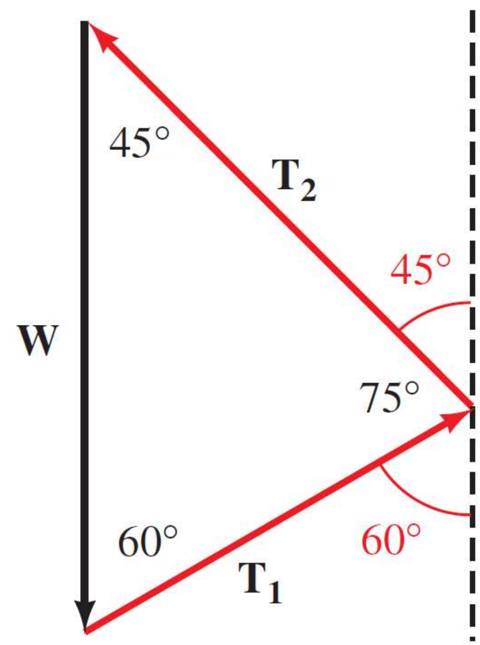

Figure 11

Example 6 – *Solution*

cont'd

Using the law of sines we have:

$$\frac{|\mathbf{T_1}|}{\sin 45^\circ} = \frac{22}{\sin 75^\circ}$$

$$|\mathbf{T_1}| = \frac{22 \sin 45^\circ}{\sin 75^\circ} = 16 \text{ lb}$$ To two significant figures

$$\frac{|\mathbf{T_2}|}{\sin 60^\circ} = \frac{22}{\sin 75^\circ}$$

$$|\mathbf{T_2}| = \frac{22 \sin 60^\circ}{\sin 75^\circ} = 20 \text{ lb}$$ To two significant figures