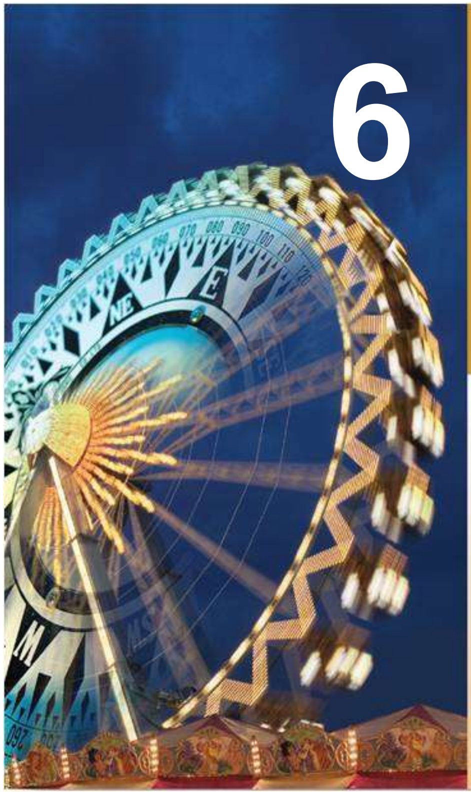

6 Equations

SECTION 6.2

More on Trigonometric Equations

Learning Objectives

1. Use an identity to solve a trigonometric equation.
2. Solve a trigonometric equation by clearing denominators.
3. Solve a trigonometric equation by squaring both sides.
4. Use a graphing calculator to approximate the solutions to a trigonometric equation.

Example 1

Solve $2 \cos x - 1 = \sec x$, if $0 \leq x < 2\pi$.

Solution:

To solve this equation as we have solved the equations in the previous section, we must write each term using the same trigonometric function.

To do so, we can use a reciprocal identity to write sec *x* in terms of cos *x*.

$$2 \cos x - 1 = \frac{1}{\cos x}$$

Example 1 – *Solution*

cont'd

To clear the equation of fractions, we multiply both sides by cos x. (Note that we must assume cos $x \neq 0$ in order to multiply both sides by it.

If we obtain solutions for which cos $x = 0$, we will have to discard them.)

$$\cos x\,(2\cos x - 1) = \frac{1}{\cos x}\cdot\cos x$$

$$2\cos^2 x - \cos x = 1$$

Example 1 – *Solution*

cont'd

We are left with a quadratic equation that we write in standard form and then solve.

$$2\cos^2 x - \cos x - 1 = 0 \qquad \text{Standard form}$$

$$(2\cos x + 1)(\cos x - 1) = 0 \qquad \text{Factor}$$

$$2\cos x + 1 = 0 \quad \text{or} \quad \cos x - 1 = 0 \qquad \text{Set each factor to 0}$$

$$\cos x = -\frac{1}{2} \quad \text{or} \quad \cos x = 1 \qquad \text{Isolate } \cos x$$

$$x = \frac{2\pi}{3}, \frac{4\pi}{3} \quad \text{or} \quad x = 0$$

The solutions are 0, $2\pi/3$, and $4\pi/3$.

Example 2

Solve $\sin 2\theta + \sqrt{2} \cos \theta = 0, 0° \leq \theta < 360°.$

Solution:

To solve this equation, both trigonometric functions must be functions of the same angle.

As the equation stands now, one angle is 2θ, while the other is θ. We can write everything as a function of θ by using the double-angle identity $\sin 2\theta = 2 \sin \theta \cos \theta$.

$$\sin 2\theta + \sqrt{2} \cos \theta = 0$$

Example 2 – *Solution*

cont'd

$$2 \sin \theta \cos \theta + \sqrt{2} \cos \theta = 0$$ Double-angle identity

$$\cos \theta (2 \sin \theta + \sqrt{2}) = 0$$ Factor out $\cos \theta$

$$\cos \theta = 0 \quad \text{or} \quad 2 \sin \theta + \sqrt{2} = 0$$ Set each factor to 0

$$\sin \theta = -\frac{\sqrt{2}}{2}$$

$$\theta = 90°, 270° \quad \text{or} \quad \theta = 225°, 315°$$

Example 5

Solve $\sin\theta - \cos\theta = 1$, if $0 \le \theta < 2\pi$.

Solution:

If we separate $\sin\theta$ and $\cos\theta$ on opposite sides of the equal sign, and then square both sides of the equation, we will be able to use an identity to write the equation in terms of one trigonometric function only.

$$\sin\theta - \cos\theta = 1$$

$$\sin\theta = 1 + \cos\theta \qquad \text{Add } \cos\theta \text{ to each side}$$

$$\sin^2\theta = (1 + \cos\theta)^2 \qquad \text{Square each side}$$

Example 5 – *Solution*

cont'd

$$\sin^2\theta = 1 + 2\cos\theta + \cos^2\theta \qquad \text{Expand } (1 + \cos\theta)^2$$

$$1 - \cos^2\theta = 1 + 2\cos\theta + \cos^2\theta \qquad \sin^2\theta = 1 - \cos^2\theta$$

$$0 = 2\cos\theta + 2\cos^2\theta \qquad \text{Standard form}$$

$$0 = 2\cos\theta(1 + \cos\theta) \qquad \text{Factor}$$

$$2\cos\theta = 0 \quad \text{or} \quad 1 + \cos\theta = 0 \qquad \text{Set factors to 0}$$

$$\cos\theta = 0 \qquad\qquad \cos\theta = -1$$

$$\theta = \pi/2,\ 3\pi/2 \quad \text{or} \quad \theta = \pi$$

Example 5 – *Solution*

cont'd

We have three possible solutions, some of which may be extraneous because we squared both sides of the equation in Step 2.

Any time we raise both sides of an equation to an even power, we have the possibility of introducing extraneous solutions.

We must check each possible solution in our original equation.

Checking $\theta = \pi/2$	*Checking* $\theta = \pi$	*Checking* $\theta = 3\pi/2$
$\sin \pi/2 - \cos \pi/2 \stackrel{?}{=} 1$	$\sin \pi - \cos \pi \stackrel{?}{=} 1$	$\sin 3\pi/2 - \cos 3\pi/2 \stackrel{?}{=} 1$

Example 5 – *Solution*

cont'd

$$1 - 0 \stackrel{?}{=} 1 \qquad 0 - (-1) \stackrel{?}{=} 1 \qquad -1 - 0 \stackrel{?}{=} 1$$

$$1 = 1 \qquad 1 = 1 \qquad -1 \neq 1$$

$\theta = \pi/2$ is a solution $\qquad \theta = \pi$ is a solution $\qquad \theta = 3\pi/2$ is not a solution

All possible solutions, except $\theta = 3\pi/2$, produce true statements when used in place of the variable in the original equation.

$\theta = 3\pi/2$ is an extraneous solution produced by squaring both sides of the equation. Our solution set is $\{\pi/2, \pi\}$.