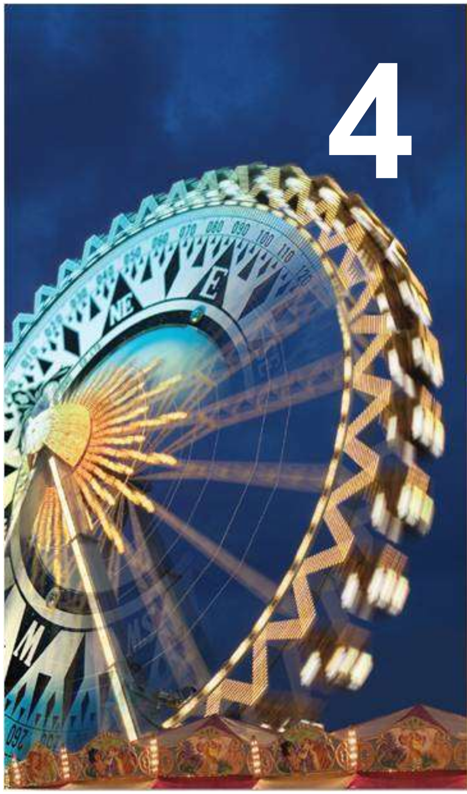

4 Graphing and Inverse Functions

SECTION 4.6

Graphing Combinations of Functions

Learning Objectives

1 Use addition of *y*-coordinates to evaluate a function.

2 Use addition of *y*-coordinates to graph a function.

Graphing Combinations of Functions

In this section, we will graph equations of the form $y = y_1 + y_2$, where y_1 and y_2 are algebraic or trigonometric functions of x. For instance, the equation $y = 1 + \sin x$ can be thought of as the sum of the two functions $y_1 = 1$ and $y_2 = \sin x$. That is,

If $\quad y_1 = 1 \quad$ and $\quad y_2 = \sin x$

then $\quad y = y_1 + y_2$

Using this kind of reasoning, the graph of $y = 1 + \sin x$ is obtained by adding each value of y_2 in $y_2 = \sin x$ to the corresponding value of y_1 in $y_1 = 1$.

Graphing Combinations of Functions

Graphically, we can show this by adding the values of y from the graph of y_2 to the corresponding values of y from the graph of y_1 (Figure1).

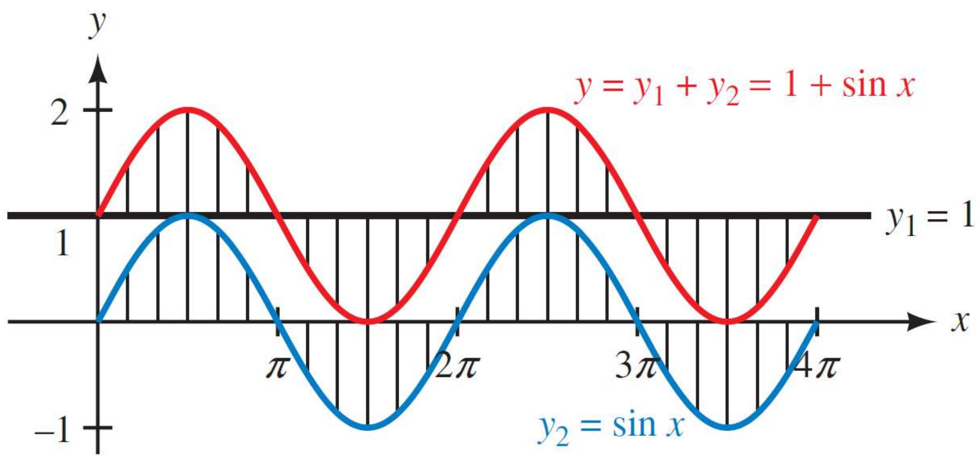

Figure1

If $y_2 > 0$, then $y_1 + y_2$ will be *above* y_1 by a distance equal to y_2. If $y_2 < 0$, then $y_1 + y_2$ will be *below* y_1 by a distance equal to the absolute value of y_2.

Graphing Combinations of Functions

Although in actual practice you may not draw in the little vertical lines we have shown here, they do serve the purpose of allowing us to visualize the idea of adding the *y*-coordinates on one graph to the corresponding *y*-coordinates on another graph.

Example 1

Graph $y = \frac{1}{3}x - \sin x$ between $x = 0$ and $x = 4\pi$.

Solution:

We can think of the equation $y = \frac{1}{3}x - \sin x$ as the sum of the equations $y_1 = \frac{1}{3}x$ and $y_2 = -\sin x$.

Graphing each of these two equations on the same set of axes and then adding the values of y_2 to the corresponding values of y_1.

Example 1 – *Solution*

cont'd

We have the graph shown in Figure 2.

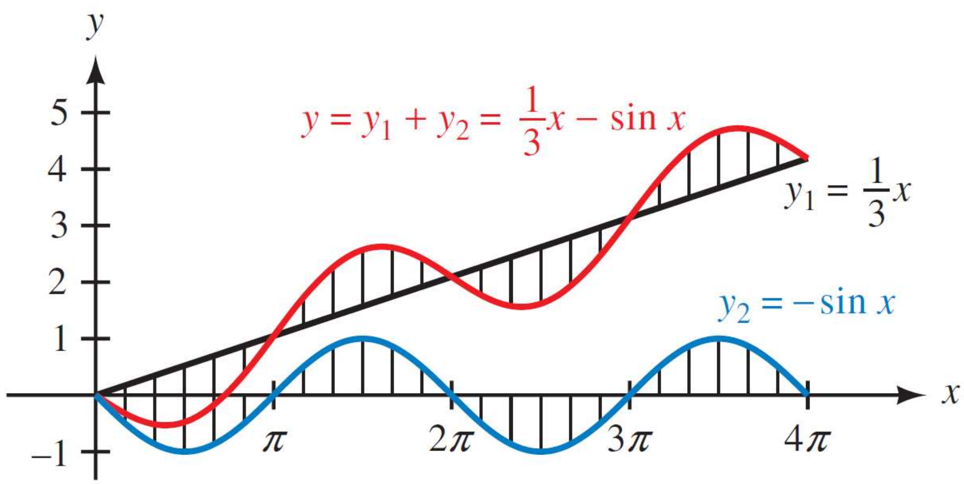

Figure 2

Graphing Combinations of Functions

One application of combining trigonometric functions can be seen with Fourier series.

Fourier series are used in physics and engineering to represent certain waveforms as an infinite sum of sine and/or cosine functions.

Example 5

The following function, which consists of an infinite number of terms, is called a Fourier series.

$$f(x) = \sin(\pi x) + \frac{\sin(3\pi x)}{3} + \frac{\sin(5\pi x)}{5} + \frac{\sin(7\pi x)}{7} + \cdots, 0 \le x < 2$$

The second partial sum of this series only includes the first two terms. Graph the second partial sum.

Solution:

We let $y_1 = \sin(\pi x)$ and $y_2 = \dfrac{\sin(3\pi x)}{3}$ and graph y_1, y_2, and $y = y_1 + y_2$.

Example 5 – *Solution*

cont'd

Figure 9 illustrates these graphs.

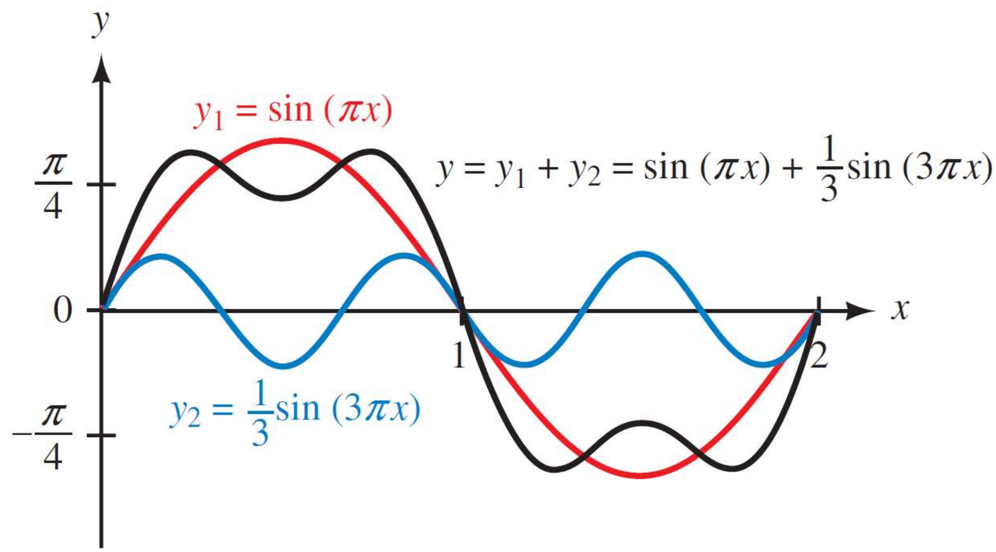

Figure 9

Graphing Combinations of Functions

The Fourier series in Example 5 is a series representation of the square wave function

$$f(x) = \begin{cases} \dfrac{\pi}{4} & 0 \le x < 1 \\ -\dfrac{\pi}{4} & 1 \le x < 2 \end{cases}$$

whose graph is shown in Figure 10.

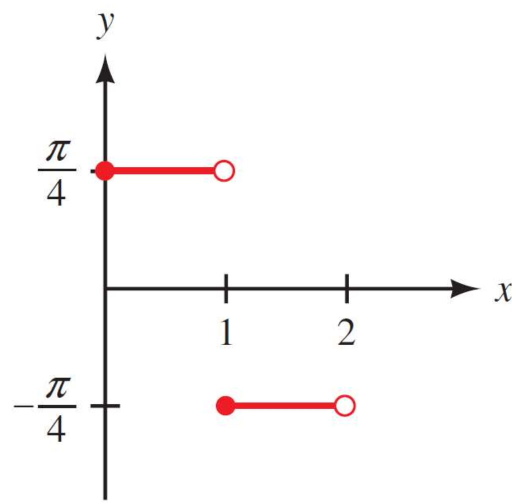

Figure 10

Graphing Combinations of Functions

In Figure 11 we have graphed the first, second, third, and fourth partial sums for the Fourier series in Example 5.

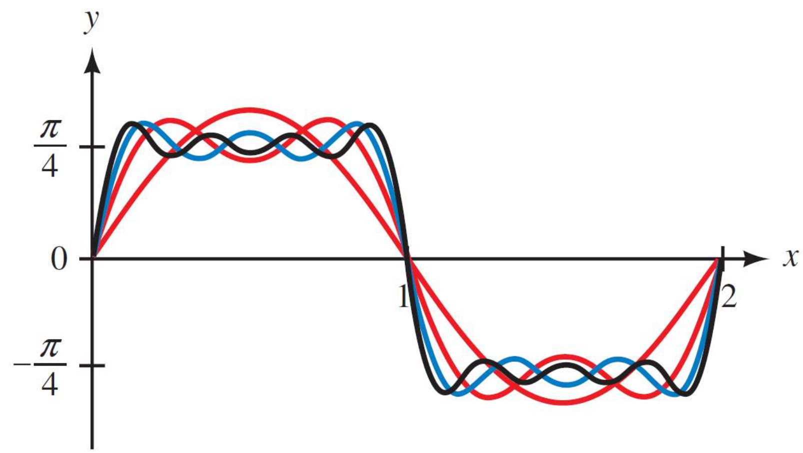

Figure 11

You can see that as we include more terms from the series, the resulting curve gives a better approximation of the square wave graph shown in Figure 10.