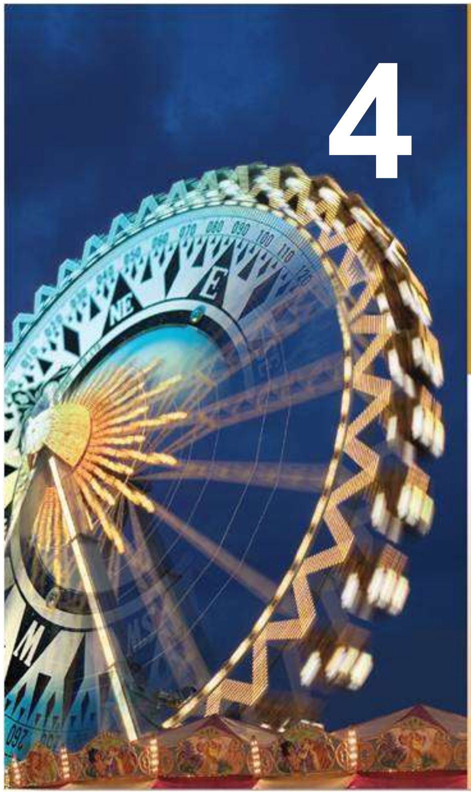

4 Graphing and Inverse Functions

SECTION 4.5

Finding an Equation from Its Graph

Learning Objectives

1. Find the equation of a line given its graph.

2. Find an equation of a sine or cosine function for a given graph.

3. Find a sinusoidal model for a real-life problem.

Example 1

Find the equation of the line shown in Figure 1.

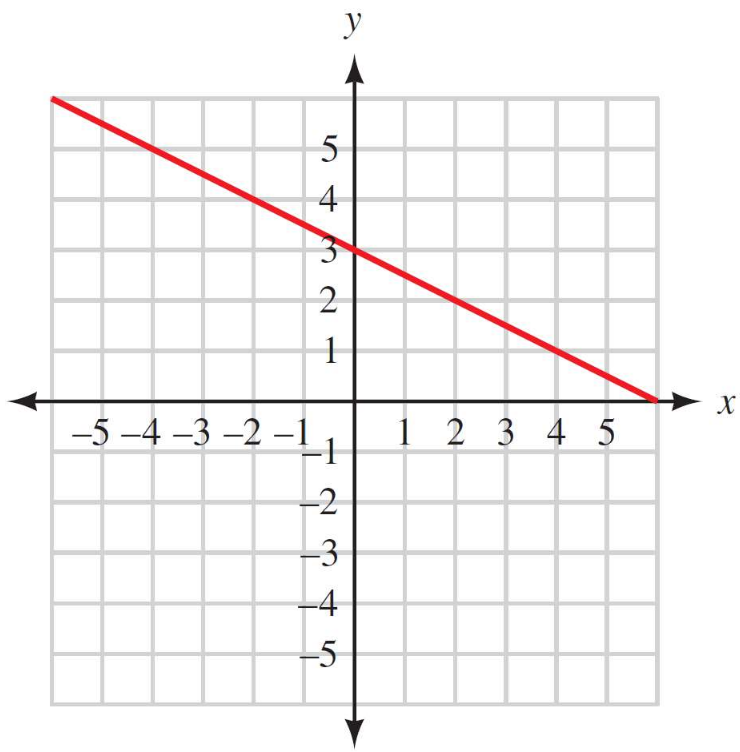

Figure 1

Example 1 – *Solution*

From algebra we know that the equation of any straight line (except a vertical one) can be written in slope-intercept form as

$$y = mx + b$$

where *m* is the slope of the line, and *b* is its *y*-intercept.

Because the line in Figure 1 crosses the *y*-axis at 3, we know the *y*-intercept *b* is 3.

To find the slope of the line, we find the ratio of the vertical change to the horizontal change between any two points on the line (sometimes called rise/run).

Example 1 – *Solution*

cont'd

From Figure 1 we see that this ratio is –1/2.

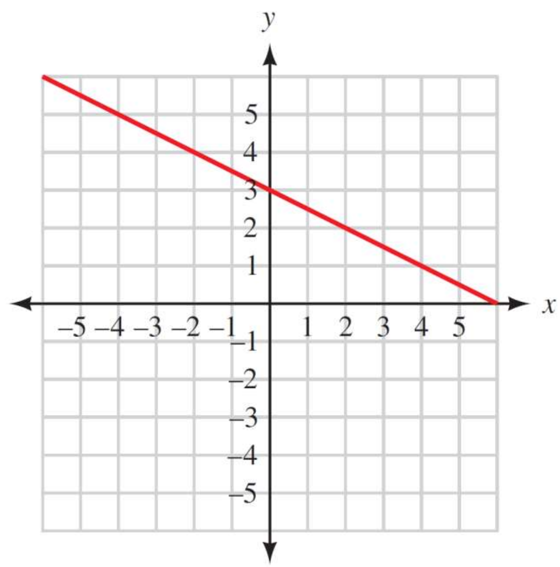

Figure 1

Therefore,
m = –1/2. The equation of our line must be

$$y = -\frac{1}{2}x + 3$$

Example 3

Find an equation of the graph shown in Figure 6.

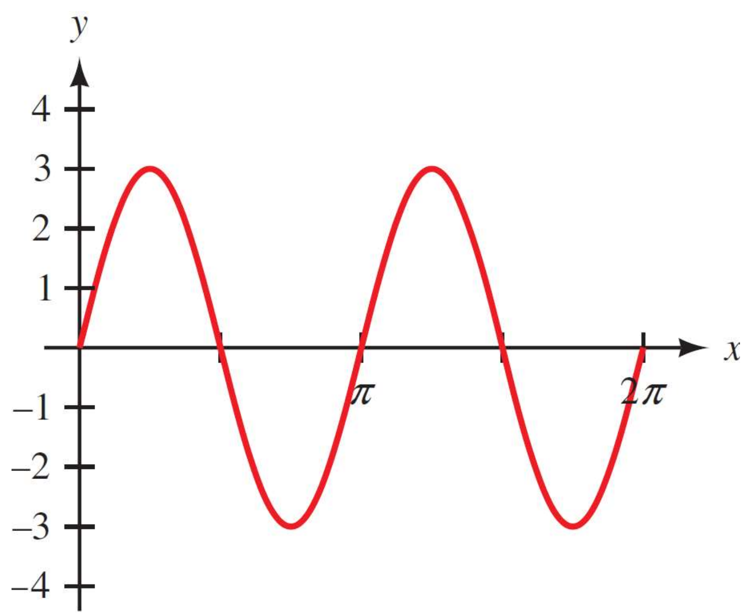

Figure 6

Example 3 – *Solution*

Again, the graph most closely matches a sine curve, so we know the equation will have the form

$$y = k + A \sin (B(x - h))$$

From Figure 6 we see that the amplitude is 3, which means that $A = 3$.

There is no horizontal shift, nor is there any vertical translation of the graph. Therefore, both h and k are 0.

Example 3 – *Solution*

cont'd

To find B, we notice that the period is π. Because the formula for the period is $2\pi/B$, we have

$$\pi = \frac{2\pi}{B}$$

which means that B is 2.

Our equation must be

$$y = 0 + 3 \sin (2(x - 0))$$

which simplifies to

$$y = 3 \sin 2x \qquad \text{for} \qquad 0 \le x \le 2\pi$$

Example 5

Find an equation of the graph shown in Figure 10.

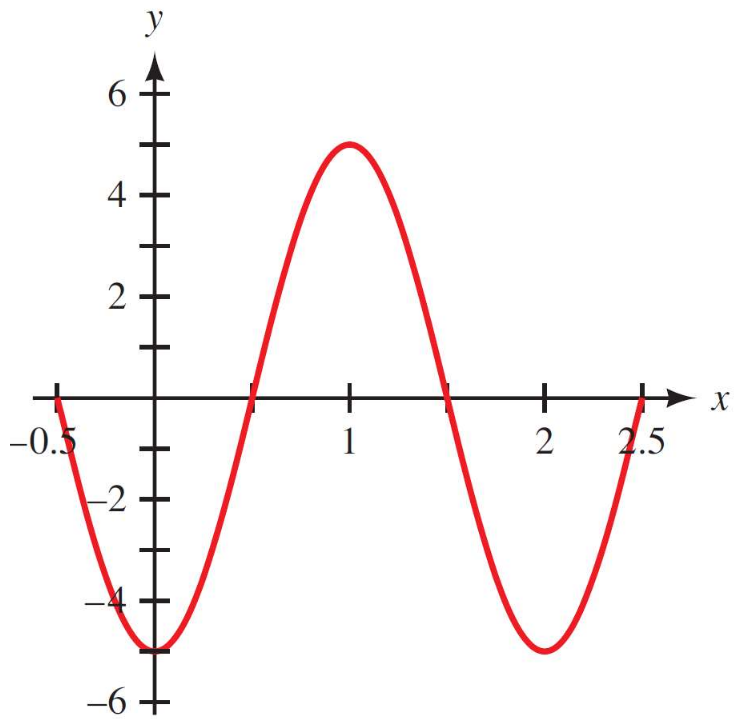

Figure 10

Example 5 – *Solution*

If we look at the graph from $x = 0$ to $x = 2$, it looks like a cosine curve that has been reflected about the x-axis. The general form for a cosine curve is

$$y = k + A\cos(B(x - h))$$

From Figure 10 we see that the amplitude is 5. Because the graph has been reflected about the x-axis, $A = -5$.

Example 5 – *Solution*

cont'd

The period is 2, giving us an equation to solve for *B*:

$$\text{Period} = \frac{2\pi}{B} = 2 \;\Rightarrow\; B = \pi$$

There is no horizontal or vertical translation of the curve (if we assume it is a cosine curve), so *h* and *k* are both 0. An equation that describes this graph is

$$y = -5 \cos \pi x \qquad \text{for} \qquad -0.5 \le x \le 2.5$$

Example 7

Table 3 shows the average monthly attendance at Lake Nacimiento in California. Find the equation of a trigonometric function to use as a model for this data.

Month	Attendance
January	6,500
February	6,600
March	15,800
April	26,000
May	38,000
June	36,000
July	31,300
August	23,500
September	12,000
October	4,000
November	900
December	2,100

Table 3

Example 7 – *Solution*

A graph of the data is shown in Figure 14, where y is the average attendance and x is the month, with $x = 1$ corresponding to January.

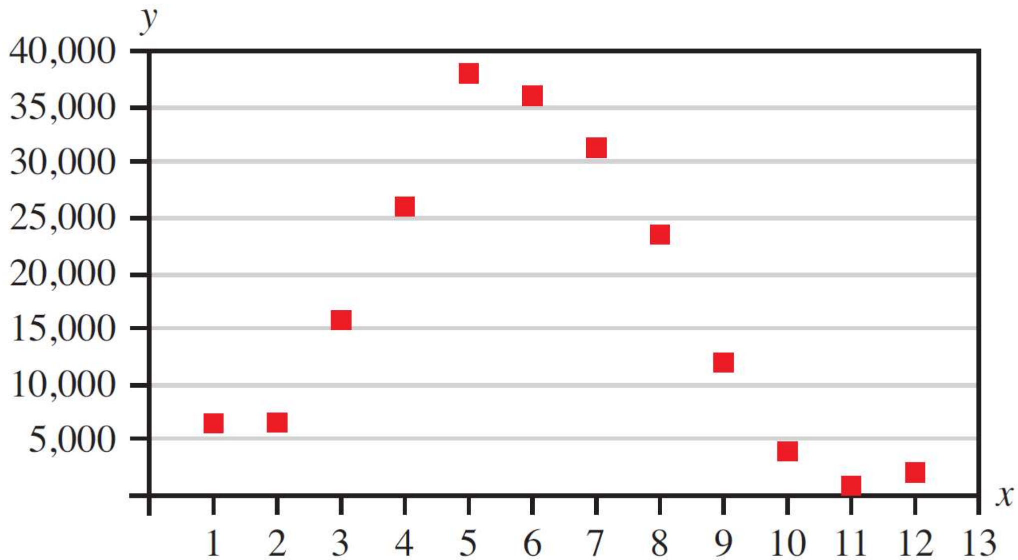

Figure 14

Example 7 – *Solution*

cont'd

We will use a cosine function to model the data. The general form is

$$y = k + A \cos (B(x - h))$$

To find the amplitude, we calculate half the difference between the maximum and minimum values.

$$A = \frac{1}{2}(38{,}000 - 900) = 18{,}550$$

For the vertical translation *k*, we average the maximum and minimum values.

$$k = \frac{1}{2}(38{,}000 + 900) = 19{,}450$$

Example 7 – *Solution*

cont'd

The horizontal distance between the maximum and minimum values is half the period. The maximum value occurs at $x = 5$ and the minimum value occurs at $x = 11$, so

$$\text{Period} = 2(11 - 5) = 12$$

Now we can find B. Because the formula for the period is $2\pi/B$, we have

$$12 = \frac{2\pi}{B} \Rightarrow B = \frac{2\pi}{12} = \frac{\pi}{6}$$

Example 7 – *Solution*

cont'd

Because a cosine cycle begins at its maximum value, the horizontal shift will be the corresponding *x*-coordinate of this point.

The maximum occurs at $x = 5$, so we have $h = 5$. Substituting the values for A, B, k, and h into the general form gives us

$$y = 19{,}450 + 18{,}550 \cos\left(\frac{\pi}{6}(x - 5)\right)$$

$$= 19{,}450 + 18{,}550 \cos\left(\frac{\pi}{6}x - \frac{5\pi}{6}\right), \qquad 0 \le x \le 12$$

Example 7 – *Solution*

cont'd

In Figure 15 we have used a graphing calculator to graph this function against the data.

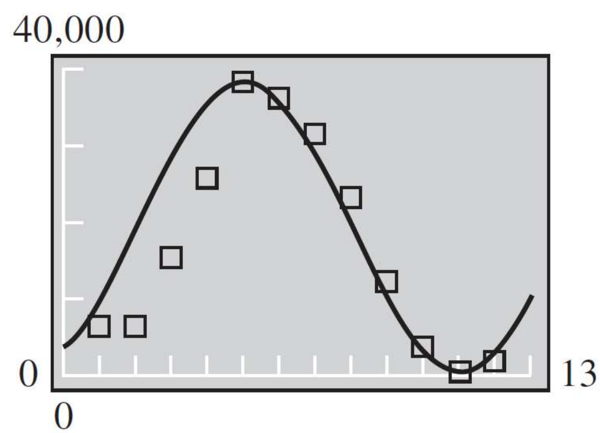

Figure 15

As you can see, the model is a good fit for the second half of the year but does not fit the data as well during the spring months.