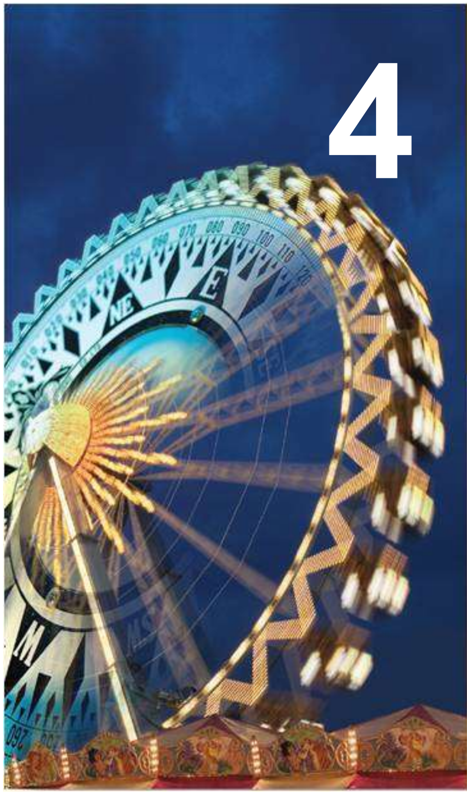

4 Graphing and Inverse Functions

SECTION 4.4

The Other Trigonometric Functions

Learning Objectives

1. Find the period and sketch the graph of a tangent function.

2. Find the period and sketch the graph of a cotangent function.

3. Find the period and sketch the graph of a secant function.

4. Find the period and sketch the graph of a cosecant function.

The Other Trigonometric Functions

The same techniques that we used to graph the sine and cosine functions can be used with the other four trigonometric functions.

Tangent and Cotangent

Example 1

Graph $y = 3 \tan x$ for $-\pi \leq x \leq \pi$.

Solution:

Although the tangent does not have a defined amplitude, we know from our work in the previous sections that the factor of 3 will triple all of the *y*-coordinates.

That is, for the same *x*, the value of *y* in $y = 3 \tan x$ will be three times the corresponding value of *y* in $y = \tan x$.

To sketch the graph of one cycle, remember that a cycle begins with an *x*-intercept, has the vertical asymptote in the middle, and ends with an *x*-intercept.

Example 1 – *Solution*

cont'd

At $x = \pi/4$ the normal y-value of 1 must be tripled, so we plot a point at $(\pi/4, 3)$. For the same reason we plot a point at $(3\pi/4, –3)$.

Figure 1 shows a complete cycle for $y = 3 \tan x$ (we have included the graph of $y = \tan x$ for comparison).

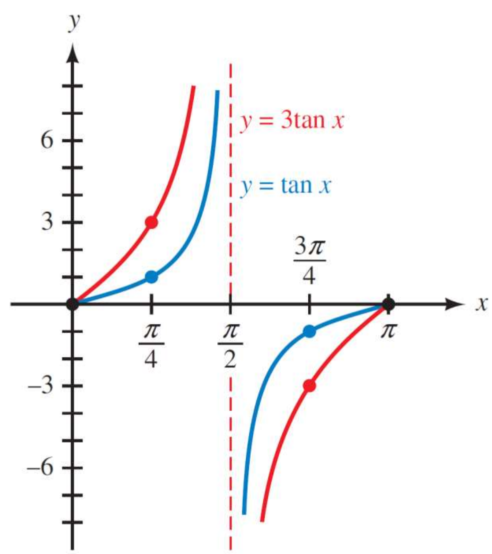

Figure 1

Example 1 – *Solution*

cont'd

The original problem asked for the graph on the interval $-\pi \le x \le \pi$. We extend the graph to the left by adding a second complete cycle. The final graph is shown in Figure 2.

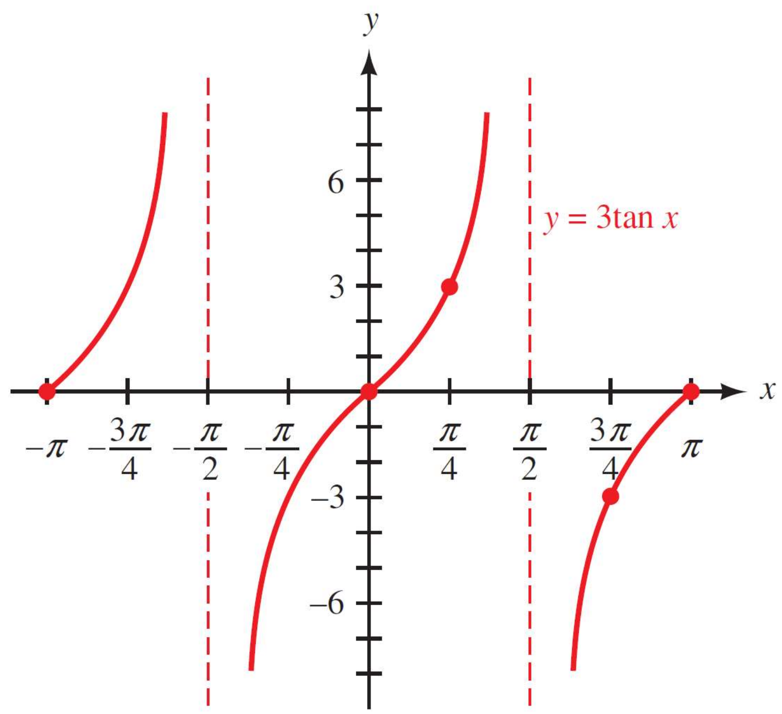

Figure 2

Example 2

Graph one complete cycle of $y = \frac{1}{2} \cot(-2x)$.

Solution:

Because the cotangent is an odd function,

$$y = \frac{1}{2} \cot(-2x) = -\frac{1}{2} \cot(2x)$$

The factor of $-\frac{1}{2}$ will halve all of the *y*-coordinates of *y* = cot (2*x*) and cause an *x*-axis reflection. In addition, there is a coefficient of 2 in the argument. To see how this will affect the period, we identify a complete cycle.

Example 2 – *Solution*

cont'd

Remember that the period of the cotangent function is π, not 2π.

One cycle: $0 \leq \text{argument} \leq \pi$

$0 \leq 2x \leq \pi$ The argument is $2x$

$0 \leq x \leq \dfrac{\pi}{2}$ Divide by 2 to isolate x

The period is $\pi/2$. Dividing this by 4 gives us $\pi/8$ so we will mark the *x*-axis in increments of $\pi/8$ starting at $x = 0$.

$$0, \quad 1 \cdot \frac{\pi}{8} = \frac{\pi}{8}, \quad 2 \cdot \frac{\pi}{8} = \frac{\pi}{4}, \quad 3 \cdot \frac{\pi}{8} = \frac{3\pi}{8}, \quad 4 \cdot \frac{\pi}{8} = \frac{\pi}{2}$$

Example 2 – *Solution*

The basic cycle of a cotangent graph begins with a vertical asymptote, has an x-intercept in the middle, and ends with another vertical asymptote.

We can sketch a frame to help plot the key points and draw the asymptotes, much as we did with the sine and cosine functions.

The difference is that the upper and lower sides of the frame do not indicate maximum and minimum values of the graph but are only used to define the position of two key points.

Example 2 – *Solution*

cont'd

Figure 3 shows the result.

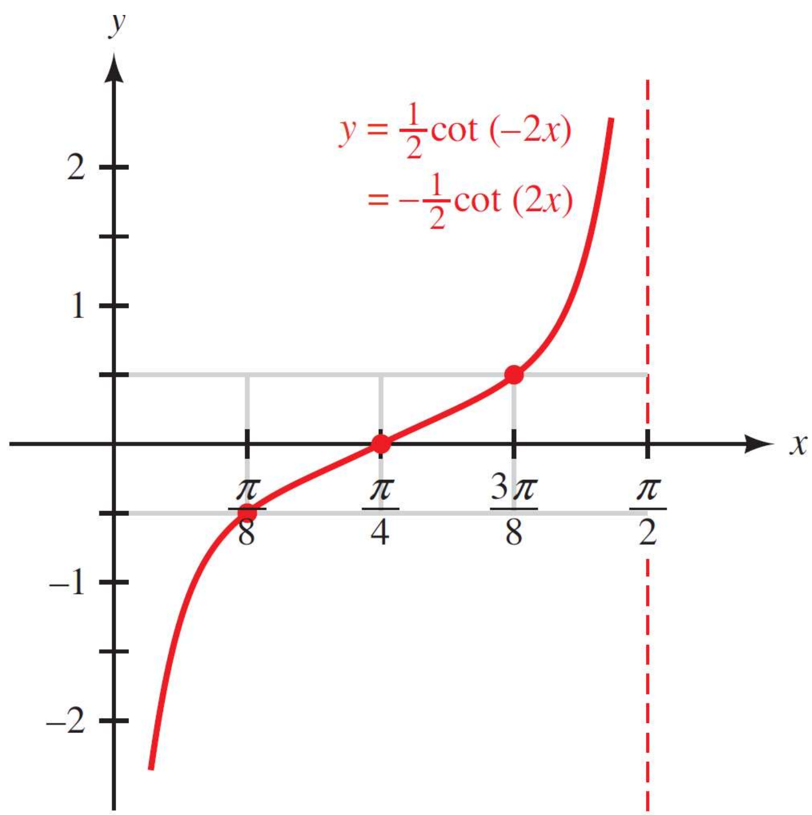

Figure 3

Tangent and Cotangent

We summarize the characteristics for the tangent and cotangent functions. The derivation of these formulas is similar, except that a period of π is used.

PERIOD AND HORIZONTAL SHIFT FOR TANGENT AND COTANGENT

If C is any real number and $B > 0$, then the graphs of $y = \tan(Bx + C)$ and $y = \cot(Bx + C)$ will have

$$\text{Period} = \frac{\pi}{B} \qquad \text{and} \qquad \text{Horizontal shift} = -\frac{C}{B}$$

Tangent and Cotangent

GRAPHING THE TANGENT AND COTANGENT FUNCTIONS

The graphs of $y = k + A\tan(B(x - h))$ and $y = k + A\cot(B(x - h))$, where $B > 0$, will have the following characteristics:

Period $= \dfrac{\pi}{B}$ Horizontal translation $= h$ Vertical translation $= k$

In addition, $|A|$ is the factor by which the basic graphs are expanded or contracted vertically. If $A < 0$ the graph will be reflected about the x-axis.

Secant and Cosecant

Secant and Cosecant

Because the secant and cosecant are reciprocals of the cosine and sine, respectively, there is a natural relationship between their graphs.

We will take advantage of this relationship to graph the secant and cosecant functions by first graphing a corresponding cosine or sine function.

Example 5

Graph one complete cycle of $y = 4 \csc x$.

Solution:

The factor of 4 will expand the graph of $y = 4 \csc x$ vertically by making all the y-coordinates four times larger.

Figure 8 shows the resulting graph of $y = \csc x$ for comparison.

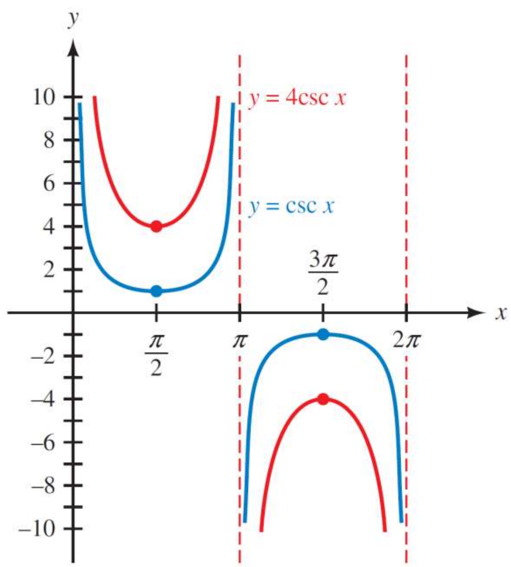

Figure 8

Example 5 – *Solution*

cont'd

In Figure 9 we have included the graph of $y = 4 \sin x$. Notice how the sine graph, in a sense, defines the behavior of the cosecant graph.

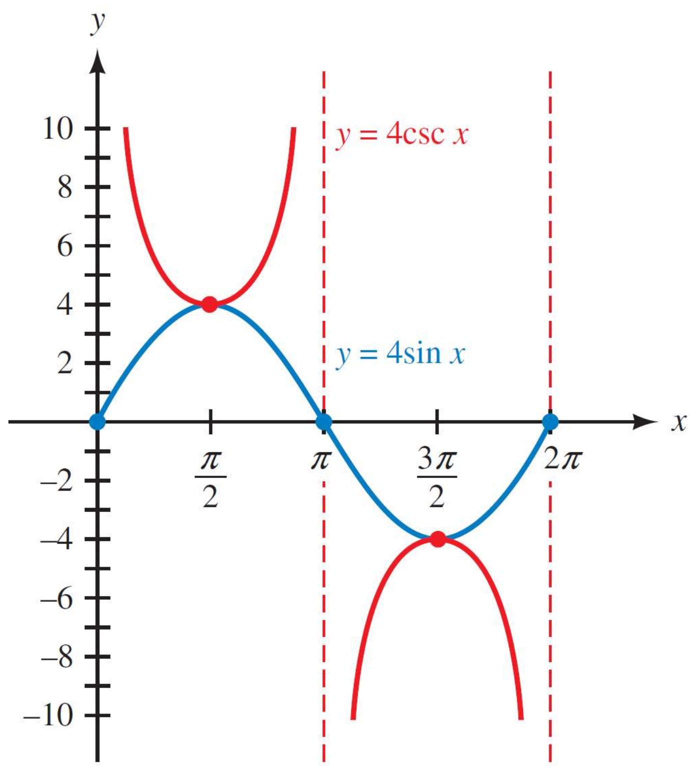

Figure 9

Example 5 – *Solution*

cont'd

The graph of $y = 4 \csc x$ has a vertical asymptote wherever $y = 4 \sin x$ crosses the x-axis (has a zero).

Furthermore, the highest and lowest points on the sine graph tell us where the two key points on the cosecant graph are.

In the remaining examples we will use a sine or cosine graph as an aid in sketching the graph of a cosecant or secant function, respectively.

Example 7

Graph one cycle of $y = -1 - 3 \csc\left(\frac{\pi x}{2} + \frac{3\pi}{4}\right)$.

Solution:

First, we sketch the graph of

$$y = -1 - 3 \sin\left(\frac{\pi x}{2} + \frac{3\pi}{4}\right)$$

There is a vertical translation of the graph of $y = \sin x$ one unit downward. The amplitude is 3 and there is a reflection about the x-axis.

Example 7 – *Solution*

cont'd

We check one cycle:

One cycle: $$0 \le \frac{\pi x}{2} + \frac{3\pi}{4} \le 2\pi$$

$$-\frac{3\pi}{4} \le \frac{\pi x}{2} \le \frac{5\pi}{4} \qquad \text{Subtract } \frac{3\pi}{4}$$

$$-\frac{3}{2} \le x \le \frac{5}{2} \qquad \text{Multiply by } \frac{2}{\pi}$$

Example 7 – *Solution*

cont'd

The horizontal shift is $-\frac{3}{2}$ and the period is 4. The graph of one cycle is shown in Figure 12.

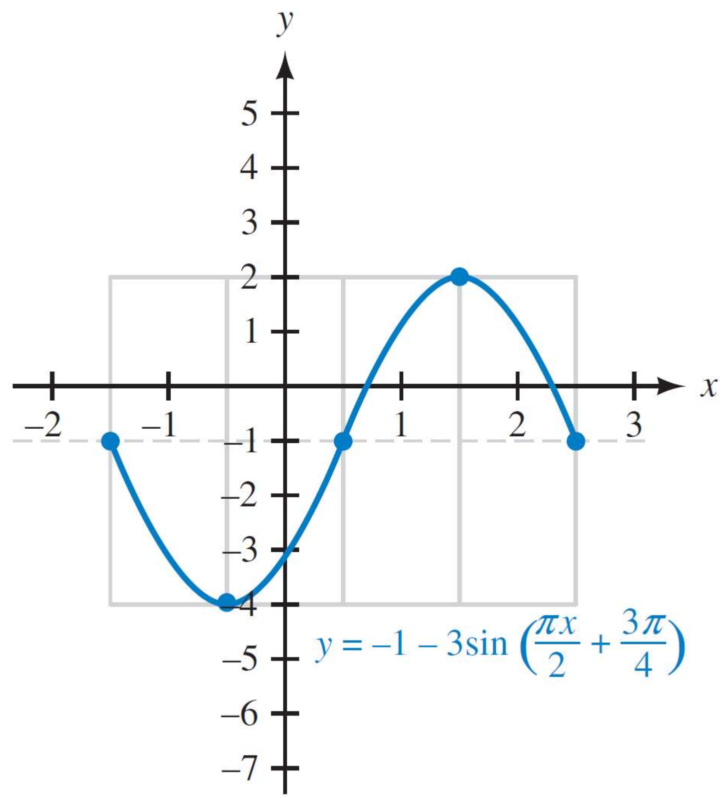

Figure 12

Example 7 – *Solution*

cont'd

Using the graph in Figure 12 as an aid, we sketch the graph of the cosecant as shown in Figure 13.

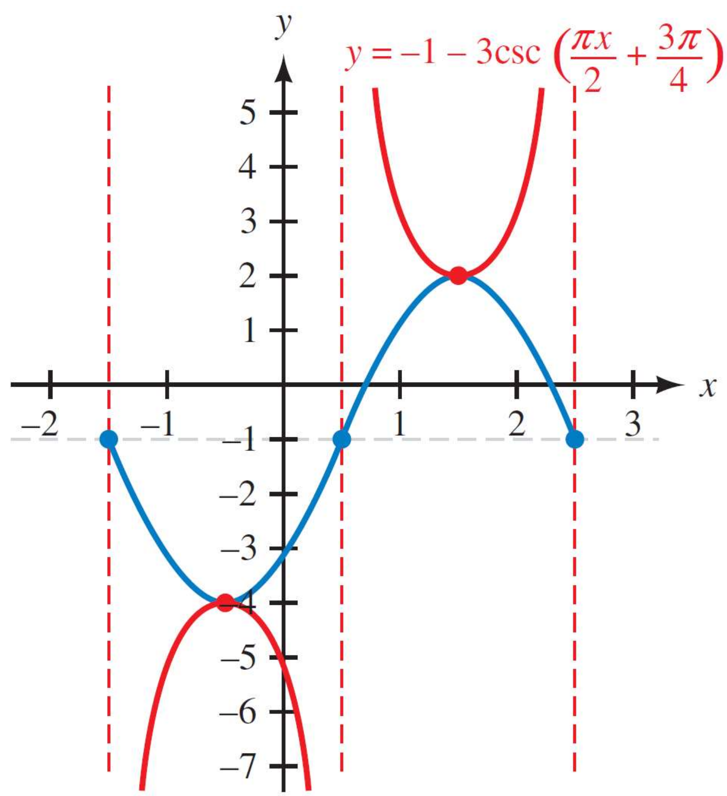

Figure 13

Secant and Cosecant

In graphing the secant and cosecant functions, the period, horizontal translation, and vertical translation are identical to those for the sine and cosine.

For this reason, we do not provide a separate summary of the characteristics for these two functions.