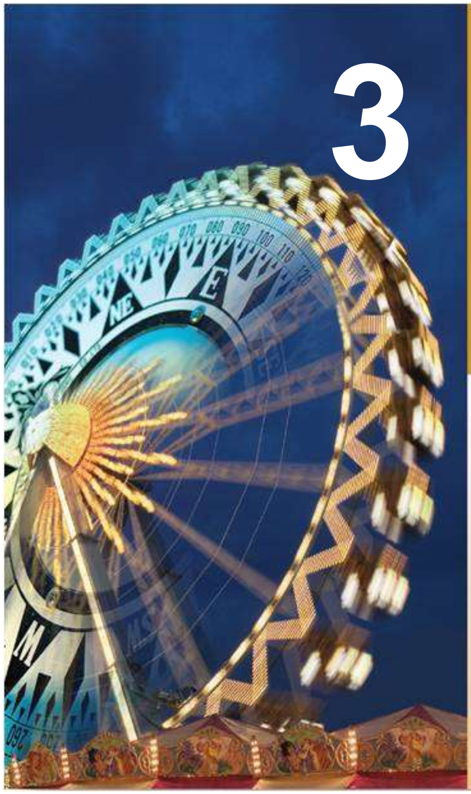

3 Radian Measure

SECTION 3.5

Velocities

Learning Objectives

1. Calculate the linear velocity of a point moving with uniform circular motion.
2. Calculate the angular velocity of a point moving with uniform circular motion.
3. Convert from linear velocity to angular velocity and vice versa.
4. Solve real-life problems involving linear and angular velocity.

Velocities

In this section, we will learn more about angular velocity and linear velocity and the relationship between them. Let's start with the formal definition for the linear velocity of a point moving on the circumference of a circle.

DEFINITION

If P is a point on a circle of radius r, and P moves a distance s on the circumference of the circle in an amount of time t, then the *linear velocity*, v, of P is given by the formula

$$v = \frac{s}{t}$$

To calculate the linear velocity, we simply divide distance traveled by time. It does not matter whether the object moves on a curve or in a straight line.

Example 1

A point on a circle travels 5 centimeters in 2 seconds. Find the linear velocity of the point.

Solution:

Substituting *s* = 5 and *t* = 2 into the equation $v = s/t$ gives us

$$v = \frac{5 \text{ cm}}{2 \text{ sec}}$$

$$= 2.5 \text{ cm/sec}$$

Velocities

DEFINITION

If P is a point moving with uniform circular motion on a circle of radius r, and the line from the center of the circle through P sweeps out a central angle θ in an amount of time t, then the *angular velocity,* ω (omega), of P is given by the formula

$$\omega = \frac{\theta}{t} \qquad \text{where } \theta \text{ is measured in radians}$$

Example 2

A point *P* on a circle rotates through $3\pi/4$ radians in 3 seconds. Give the angular velocity of *P*.

Solution:

Substituting $\theta = 3\pi/4$ and $t = 3$ into the equation $\omega = \theta/t$ gives us

$$\omega = \frac{3\pi/4 \text{ rad}}{3 \text{ sec}}$$

$$= \frac{\pi}{4} \text{ rad/sec}$$

The Relationship Between the Two Velocities

The Relationship Between the Two Velocities

$$\text{If} \qquad s = r\theta$$

$$\text{then} \qquad \frac{s}{t} = \frac{r\theta}{t}$$

$$\frac{s}{t} = r\frac{\theta}{t}$$

$$v = r\omega$$

Linear velocity is the product of the radius and the angular velocity. Just as with arc length, this relationship implies that the linear velocity is proportional to the angular velocity because *r* would be constant for a given circle.

The Relationship Between the Two Velocities

LINEAR AND ANGULAR VELOCITY

If a point is moving with uniform circular motion on a circle of radius r, then the linear velocity v and angular velocity ω of the point are related by the formula

$$v = r\omega$$

Example 5

A phonograph record is turning at 45 revolutions per minute (rpm). If the distance from the center of the record to a point on the edge of the record is 3 inches, find the angular velocity and the linear velocity of the point in feet per minute.

Solution:

The quantity 45 revolutions per minute is another way of expressing the rate at which the point on the record is moving.

We can obtain the angular velocity from it by remembering that one complete revolution is equivalent to 2π radians.

Example 5 – *Solution*

cont'd

Therefore,

$$\omega = 45 \text{ rpm}$$

$$= \frac{45 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}}$$

$$= 90\pi \text{ rad/min}$$

Because one revolution is equivalent to 2π radians, the fraction

$$\frac{2\pi \text{ rad}}{1 \text{ rev}}$$

is just another form of the number 1. We call this kind of fraction a *conversion factor*.

Example 5 – *Solution*

cont'd

Notice how the units of revolutions divide out in much the same way that common factors divide out when we reduce fractions to lowest terms.

The conversion factor allows us to convert from revolutions to radians by dividing out revolutions.

To find the linear velocity, we multiply ω by the radius.

$$v = r\omega$$

$$= (3 \text{ in.})(90\pi \text{ rad/min})$$

Example 5 – *Solution*

cont'd

$$
\begin{aligned}
&= 270\pi \text{ in./min} && \text{Exact value} \\
&\approx 270(3.14) && \text{Approximate value} \\
&= 848 \text{ in./min} && \text{To three significant digits}
\end{aligned}
$$

To convert 848 inches per minute to feet per minute, we use another conversion factor relating feet to inches. Here is our work:

$$
\begin{aligned}
848 \text{ in./min} &= \frac{848 \cancel{\text{in.}}}{1 \text{ min}} \cdot \frac{1 \text{ ft}}{12 \cancel{\text{in.}}} \\
&= \frac{848}{12} \text{ ft/min} \\
&\approx 70.7 \text{ ft/min}
\end{aligned}
$$

Example 6

The Ferris wheel is shown in Figure 2. If the diameter of the wheel is 250 feet, the distance from the ground to the bottom of the wheel is 14 feet, and one complete revolution takes 20 minutes, find the following.

a. The linear velocity, in miles per hour, of a person riding on the wheel.

b. The height of the rider in terms of the time t, where t is measured in minutes.

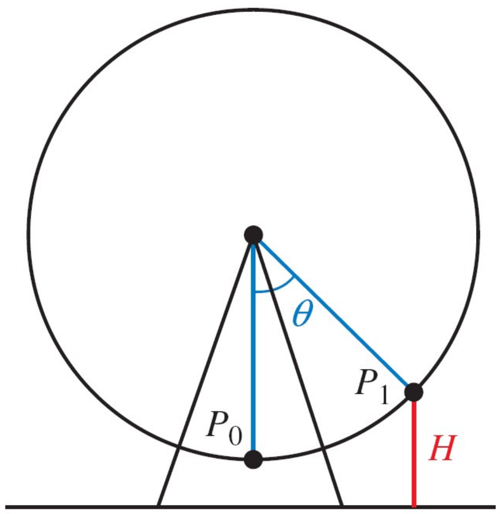

Figure 2

Example 6(a) – *Solution*

We found the angular velocity in the introduction to this section. It is

$$\omega = \frac{\pi}{10} \text{ rad/min}$$

Next, we use the formula $v = r\omega$ to find the linear velocity. That is, we multiply the angular velocity by the radius to find the linear velocity.

$$\begin{aligned} v &= r\omega \\ &= (125 \text{ ft})\left(\frac{\pi}{10} \text{ rad/min}\right) \\ &\approx 39.27 \text{ ft/min (intermediate answer)} \end{aligned}$$

Example 6(a) – *Solution*

cont'd

To convert to miles per hour, we use the facts that there are 60 minutes in 1 hour and 5,280 feet in 1 mile.

$$39.27 \text{ ft/min} = \frac{39.27 \cancel{\text{ft}}}{1 \cancel{\text{min}}} \cdot \frac{60 \cancel{\text{min}}}{1 \text{ hr}} \cdot \frac{1 \text{ mi}}{5{,}280 \cancel{\text{ft}}}$$

$$= \frac{(39.27)(60) \text{ mi}}{5{,}280 \text{ hr}}$$

$$\approx 0.45 \text{ mi/hr}$$

Example 6(b) – *Solution*

cont'd

Suppose the person riding on the wheel is at position P_1 as shown in Figure 2.

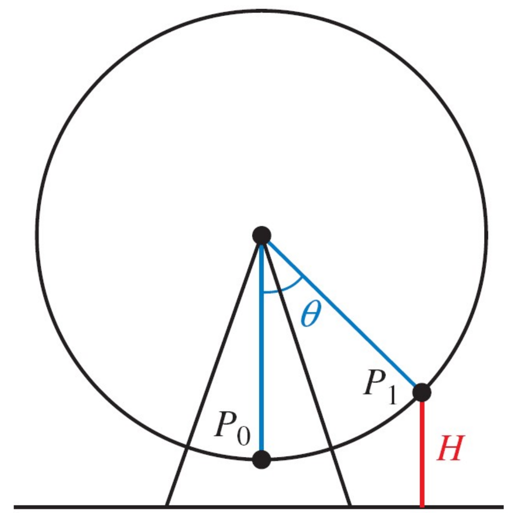

Figure 2

Based on our work in Example 5 of Section 2.3, the height, *H*, of the rider can be found from the central angle θ using the equation

$$H = 139 - 125 \cos \theta$$

Example 6(b) – *Solution*

cont'd

Because the angular velocity of the wheel is $\omega = \pi/10$ radians per minute, then assuming θ is measured in radians we have

$$\omega = \frac{\theta}{t} \qquad \text{Definition of angular velocity}$$

$$\theta = \omega t \qquad \text{Multiply both sides by } t$$

$$\theta = \frac{\pi}{10} t \qquad \text{Substitute } \omega = \pi/10$$

So

$$H = 139 - 125 \cos\left(\frac{\pi}{10} t\right)$$

This equation gives us the height of the rider at any time *t*, where *t* is measured in minutes from the beginning of the ride.