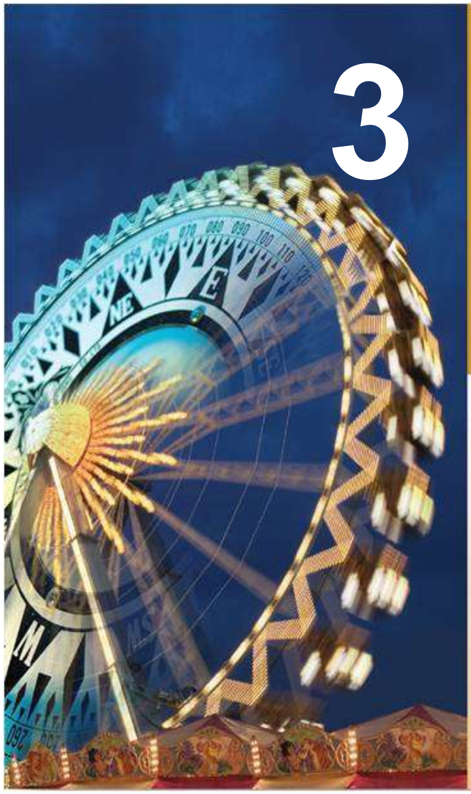

3 Radian Measure

SECTION 3.4

Arc Length and Area of a Sector

Learning Objectives

1 Calculate the arc length for a central angle.

2 Calculate the area of a sector formed by a central angle.

3 Solve a real-life problem involving arc length.

4 Solve a real-life problem involving sector area.

Arc Length and Area of a Sector

We have known that if a central angle θ, measured in radians, in a circle of radius *r* cuts off an arc of length *s*, then the relationship between *s*, *r*, and θ can be written as

$$\theta = \frac{s}{r}.$$

Figure 1 illustrates this. If we multiply both sides of this equation by *r*, we will obtain the equation that gives arc length *s* in terms of *r* and θ.

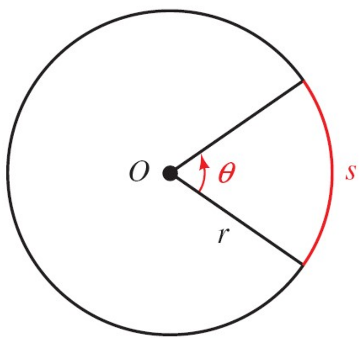

Figure 1

Arc Length and Area of a Sector

ARC LENGTH

If θ (in radians) is a central angle in a circle with radius r, then the length of the arc cut off by θ is given by

$$s = r\theta \qquad (\theta \text{ in radians})$$

Because *r* would be constant for a given circle, this formula tells us that the arc length is proportional to the central angle. An angle twice as large would cut off an arc twice as long.

Example 1

Give the length of the arc cut off by a central angle of 2 radians in a circle of radius 4.3 inches.

Solution:

We have $\theta = 2$ and $r = 4.3$ inches. Applying the formula $s = r\theta$ gives us

$$
\begin{aligned}
s &= r\theta \\
&= 4.3(2) \\
&= 8.6 \text{ inches}
\end{aligned}
$$

Example 1 – *Solution*

Figure 2 illustrates this example.

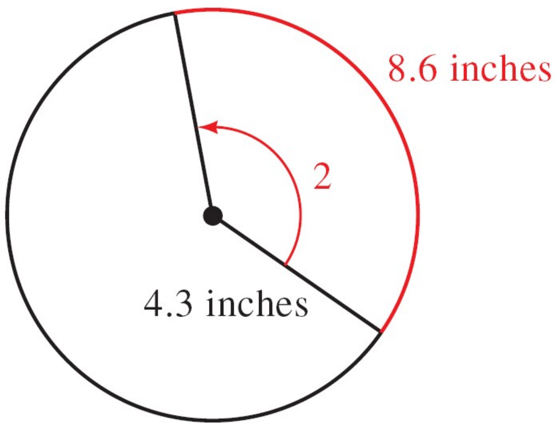

Figure 2

Arc Length and Area of a Sector

If we are working with relatively small central angles in circles with large radii, we can use the length of the intercepted arc to approximate the length of the associated chord.

For example, Figure 5 shows a central angle of 1° in a circle of radius 1,800 feet, along with the arc and chord cut off by 1°. (Figure 5 is not drawn to scale.)

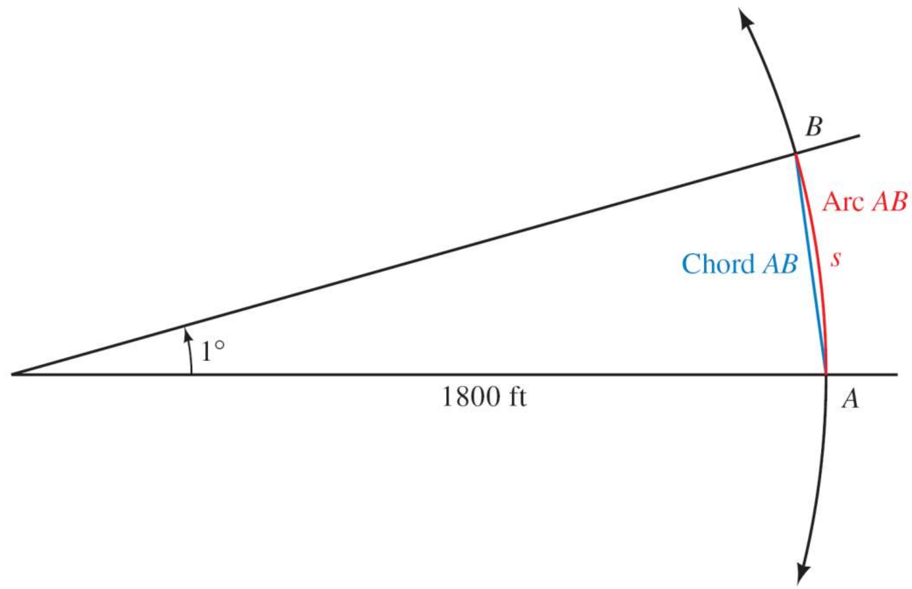

Figure 5

Arc Length and Area of a Sector

To find the length of arc *AB*, we convert θ to radians by multiplying by $\pi/180$. Then we apply the formula $s = r\theta$.

$$s = r\theta = 1{,}800(1)\left(\frac{\pi}{180}\right) = 10\pi \approx 31 \text{ ft}$$

If we had carried out the calculation of arc *AB* to six significant digits, we would have obtained $s = 31.4159$.

The length of the chord *AB* is 31.4155 to six significant digits.

Example 4

A person standing on the earth notices that a 747 Jumbo Jet flying overhead subtends an angle of 0.45°. If the length of the jet is 230 feet, find its altitude to the nearest thousand feet.

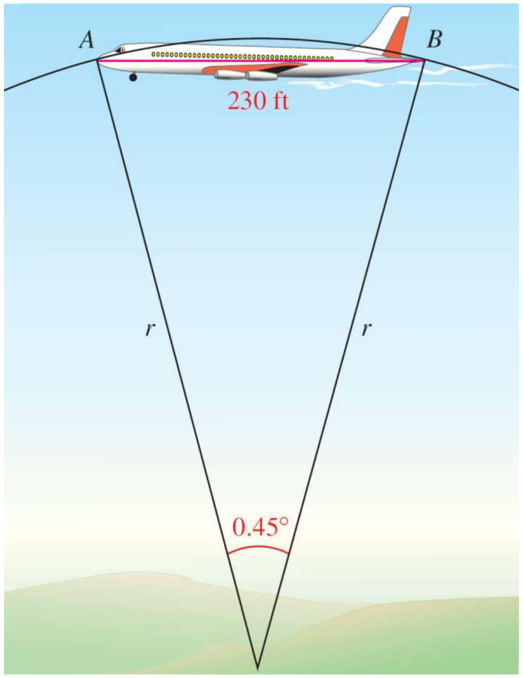

Figure 6

Example 4 – *Solution*

Figure 6 is a diagram of the situation. Because we are working with a relatively small angle in a circle with a large radius, we use the length of the airplane (chord *AB* in Figure 6) as an approximation of the length of the arc *AB*, and *r* as an approximation for the altitude of the plane.

Because $s = r\theta$ then $r = \dfrac{s}{\theta}$

$$= \frac{230}{(0.45)(\pi/180)}$$

We multiply 0.45° by $\pi/180$ to change to radian measure

$$= \frac{230(180)}{(0.45)(\pi)}$$

$$= 29{,}000 \text{ ft}$$

To the nearest thousand feet

Area of a Sector

Area of a Sector

Next we want to derive the formula for the area of the sector formed by a central angle θ (Figure 7).

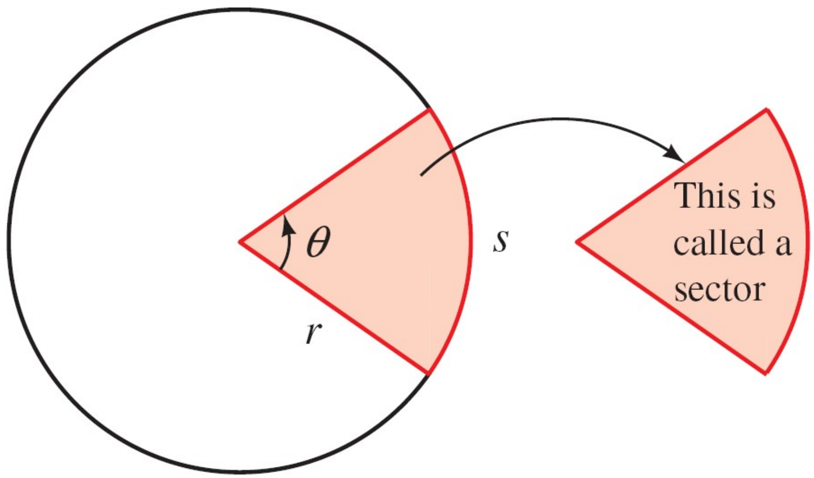

Figure 7

Area of a Sector

AREA OF A SECTOR

If θ (in radians) is a central angle in a circle with radius r, then the area of the sector (Figure 8) formed by angle θ is given by

$$A = \frac{1}{2}r^2\theta \qquad (\theta \text{ in radians})$$

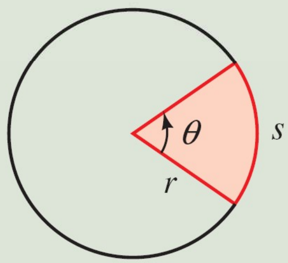

Figure 8

Example 5

Find the area of the sector formed by a central angle of 1.4 radians in a circle of radius 2.1 meters.

Solution:

We have r = 2.1 meters and θ = 1.4. Applying the formula for A gives us

$$A = \frac{1}{2}r^2\theta$$

$$= \frac{1}{2}(2.1)^2(1.4)$$

$$= 3.1 \text{ m}^2$$ To the nearest tenth

Example 7

A lawn sprinkler located at the corner of a yard is set to rotate through 90° and project water out 30.0 feet. To three significant digits, what area of lawn is watered by the sprinkler?

Solution:

We have $\theta = 90° = \pi/2 \approx 1.57$ radians and $r = 30.0$ feet. Figure 9 illustrates this example.

$$A = \frac{1}{2}r^2\theta \approx \frac{1}{2}(30.0)^2(1.57)$$

$$= 707 \text{ ft}^2$$

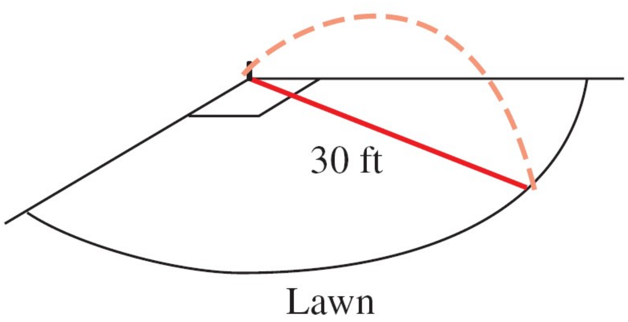

Figure 9