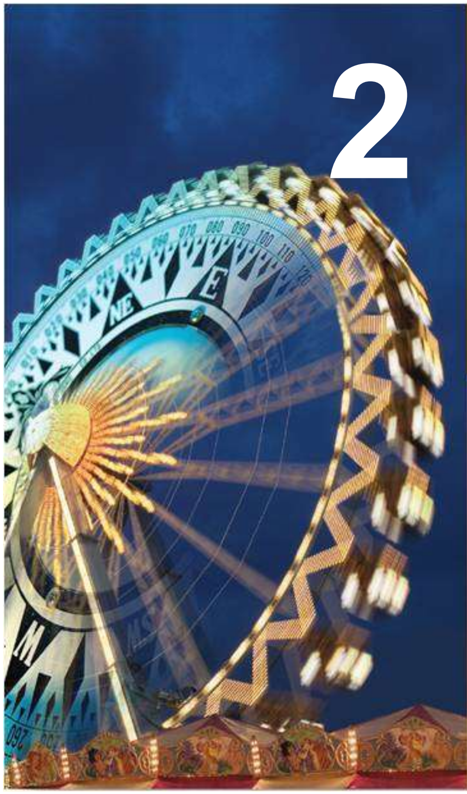

2 Right Triangle Trigonometry

SECTION 2.4 Applications

Learning Objectives

1 Correctly interpret a bearing.

2 Solve a real-life problem involving an angle of elevation or depression.

3 Solve a real-life problem involving bearing.

4 Solve an applied problem using right triangle trigonometry.

Applications

In this section we will see how this is done by looking at a number of applications of right triangle trigonometry.

Example 1

The two equal sides of an isosceles triangle are each 24 centimeters. If each of the two equal angles measures 52°, find the length of the base and the altitude.

Solution:

An isosceles triangle is any triangle with two equal sides. The angles opposite the two equal sides are called the base angles, and they are always equal.

Figure 1 shows a picture of our isosceles triangle.

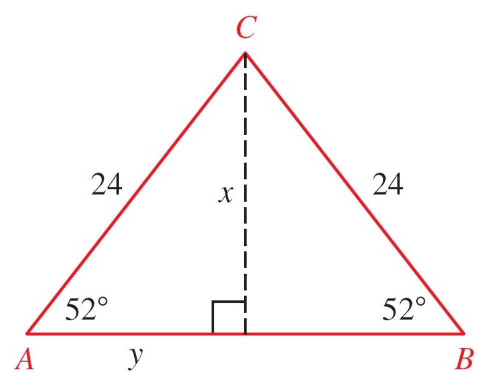

Figure 1

Example 1 – *Solution*

cont'd

We have labeled the altitude *x*. We can solve for *x* using a sine ratio.

If $\sin 52° = \dfrac{x}{24}$

then $x = 24 \sin 52°$

$= 24(0.7880)$

$= 19 \text{ cm}$ Rounded to two significant digits

Example 1 – *Solution*

cont'd

We have labeled half the base with *y*. To solve for *y*, we can use a cosine ratio.

$$\text{If} \qquad \cos 52^\circ = \frac{y}{24}$$

$$\text{then} \qquad y = 24 \cos 52^\circ$$

$$= 24(0.6157)$$

$$= 15 \text{ cm} \qquad \text{To two significant digits}$$

The base is $2y = 2(15) = 30$ cm.

Applications

For our next applications, we need the following definition.

DEFINITION

An angle measured from the horizontal up is called an *angle of elevation*. An angle measured from the horizontal down is called an *angle of depression* (Figure 2).

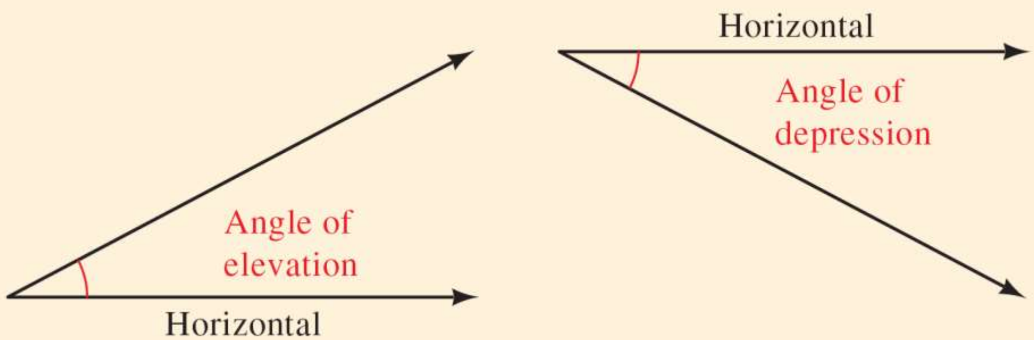

Figure 2

These angles of elevation and depression are always considered positive angles.

Applications

Also, if an observer positioned at the vertex of the angle views an object in the direction of the nonhorizontal side of the angle, then this side is sometimes called the *line of sight* of the observer.

Example 2

If a 75.0-foot flagpole casts a shadow 43.0 feet long, to the nearest 10 minutes what is the angle of elevation of the sun from the tip of the shadow?

Solution:

We begin by making a diagram of the situation (Figure 3).

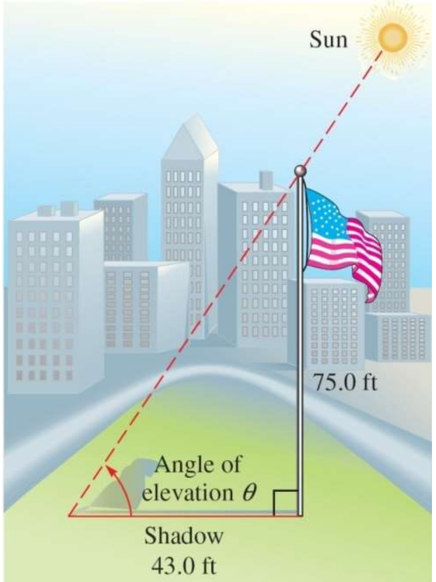

Figure 3

Example 2 – *Solution*

cont'd

If we let θ = the angle of elevation of the sun, then

$$\tan \theta = \frac{75.0}{43.0}$$

$$\tan \theta = 1.7442$$

which means $\theta = \tan^{-1}(1.7442) = 60° \ 10'$ to the nearest 10 minutes.

Applications

Our next applications are concerned with what is called the *bearing of a line*. It is used in navigation and surveying.

DEFINITION

The *bearing of a line* l is the acute angle formed by the north–south line and the line l. The notation used to designate the bearing of a line begins with N or S (for north or south), followed by the number of degrees in the angle, and ends with E or W (for east or west).

Example 5

San Luis Obispo, California, is 12 miles due north of Grover Beach. If Arroyo Grande is 4.6 miles due east of Grover Beach, what is the bearing of San Luis Obispo from Arroyo Grande?

Solution:

We are looking for the bearing of San Luis Obispo from Arroyo Grande, so we will put our N-S-E-W system on Arroyo Grande (Figure 8).

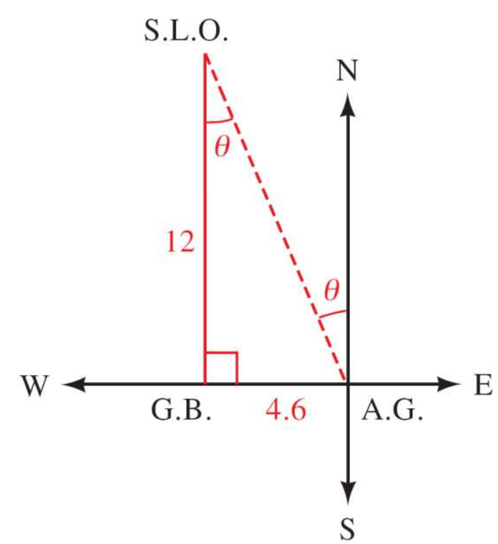

Figure 8

Example 5 – *Solution*

cont'd

We have known that when two parallel lines are crossed by a transversal, alternate interior angles are equal.

Now we can solve for θ using the tangent ratio.

$$\tan\theta = \frac{4.6}{12}$$

$$\tan\theta = 0.3833$$

$$\theta = \tan^{-1}(0.3833) = 21^\circ$$ To the nearest degree

The bearing of San Luis Obispo from Arroyo Grande is N 21° W.

Example 8

A helicopter is hovering over the desert when it develops mechanical problems and is forced to land. After landing, the pilot radios his position to a pair of radar stations located 25 miles apart along a straight road running north and south.

The bearing of the helicopter from one station is N 13° E, and from the other it is S 19° E. After doing a few trigonometric calculations, one of the stations instructs the pilot to walk due west for 3.5 miles to reach the road. Is this information correct?

Example 8 – *Solution*

Figure 11 is a three-dimensional diagram of the situation.

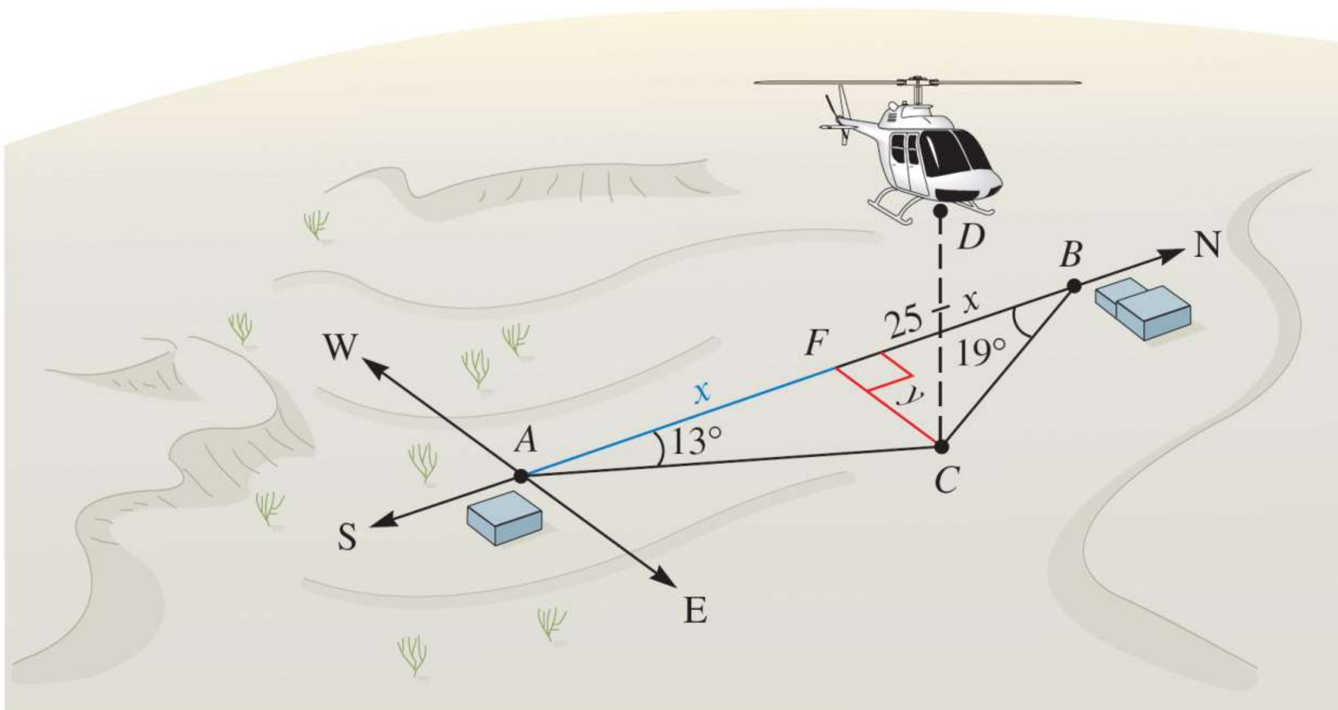

Figure 11

Example 8 – *Solution*

cont'd

The helicopter is hovering at point *D* and lands at point *C*. The radar stations are at *A* and *B*, respectively.

Because the road runs north and south, the shortest distance from *C* to the road is due west of *C* toward point *F*.

To see if the pilot has the correct information, we must find *y*, the distance from *C* to *F*.

The radar stations are 25 miles apart, thus $AB = 25$. If we let $AF = x$, then $FB = 25 - x$. If we use cotangent ratios in triangles *AFC* and *BFC*, we will save ourselves some work.

Example 8 – *Solution*

cont'd

In triangle AFC $\qquad \cot 13^\circ = \dfrac{x}{y}$

so $\qquad x = y \cot 13^\circ$

In triangle BFC $\qquad \cot 19^\circ = \dfrac{25 - x}{y}$

so $\qquad 25 - x = y \cot 19^\circ$

Solving this equation for *x* we have.

$$-x = -25 + y \cot 19^\circ \qquad \text{Add } -25 \text{ to each side}$$

$$x = 25 - y \cot 19^\circ \qquad \text{Multiply each side by } -1$$

Example 8 – *Solution*

cont'd

Next, we set our two values of *x* equal to each other.

$$x = x$$

$$y \cot 13° = 25 - y \cot 19°$$

$$y \cot 13° + y \cot 19° = 25 \qquad \text{Add } y \cot 19° \text{ to each side}$$

$$y(\cot 13° + \cot 19°) = 25 \qquad \text{Factor } y \text{ from each term}$$

$$y = \frac{25}{\cot 13° + \cot 19°} \qquad \text{Divide by the coefficient of } y$$

Example 8 – *Solution*

cont'd

$$= \frac{25}{4.3315 + 2.9042}$$

$$= \frac{25}{7.2357}$$

$$= 3.5 \text{ mi}$$ To two significant digits

The information given to the pilot is correct.