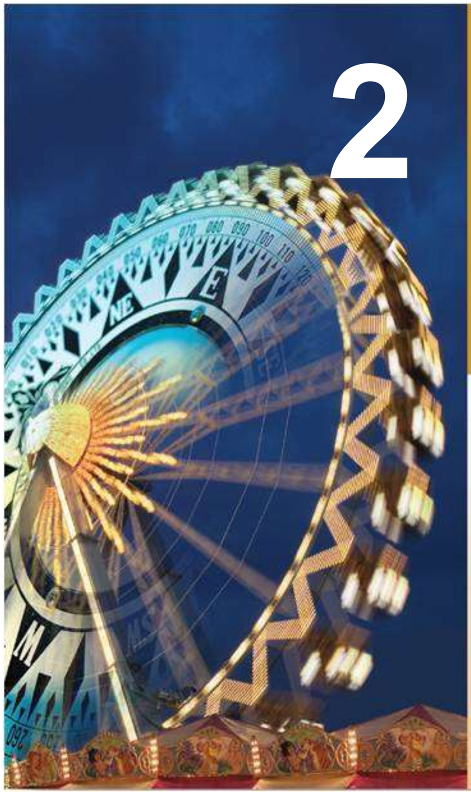

2 Right Triangle Trigonometry

SECTION 2.3

Solving Right Triangles

Learning Objectives

1. Determine the number of significant digits in a value.
2. Solve a right triangle for a missing side.
3. Solve a right triangle for a missing angle.
4. Solve a real-life problem using right triangle trigonometry.

Solving Right Triangles

DEFINITION

The number of *significant digits* (or figures) in a number is found by counting all the digits from left to right beginning with the first nonzero digit on the left. When no decimal point is present, trailing zeros are not considered significant.

According to this definition,

0.042 has two significant digits
0.005 has one significant digit
20.5 has three significant digits
6.000 has four significant digits
9,200. has four significant digits
700 has one significant digit

Solving Right Triangles

The relationship between the accuracy of the sides of a triangle and the accuracy of the angles in the same triangle is shown in Table 1.

Accuracy of Sides	Accuracy of Angles
Two significant digits	Nearest degree
Three significant digits	Nearest 10 minutes or tenth of a degree
Four significant digits	Nearest minute or hundredth of a degree

Table 1

We are now ready to use Definition II to solve right triangles. We solve a right triangle by using the information given about it to find all of the missing sides and angles.

Solving Right Triangles

In all the examples, we will assume that *C* is the right angle in all of our right triangles, unless otherwise noted.

Unless stated otherwise, we round our answers so that the number of significant digits in our answers matches the number of significant digits in the least significant number given in the original problem.

Also, we round our answers only and not any of the numbers in the intermediate steps.

Solving Right Triangles

Finally, we are showing the values of the trigonometric functions to four significant digits simply to avoid cluttering the page with long decimal numbers.

In Example 2, we are given two sides and asked to find the remaining parts of a right triangle.

Example 2

In right triangle *ABC*, $a = 2.73$ and $b = 3.41$. Find the remaining side and angles.

Solution:

Figure 2 is a diagram of the triangle.

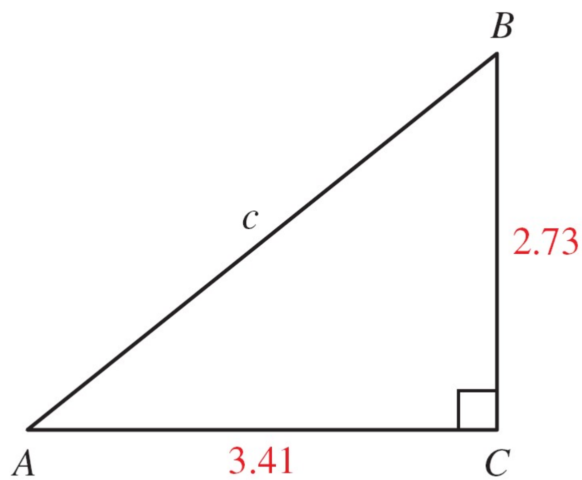

Figure 2

Example 2 – *Solution*

cont'd

We can find A by using the formula for tan A.

$$\tan A = \frac{a}{b}$$

$$= \frac{2.73}{3.41}$$

$$\tan A = 0.8006$$

Now, to find A, we use a calculator.

$$A = \tan^{-1}(0.8006) = 38.7°$$

Example 2 – *Solution*

cont'd

Next we find *B*.

$$B = 90.0° - A$$

$$= 90.0° - 38.7°$$

$$B = 51.3°$$

Notice we are rounding each angle to the nearest tenth of a degree since the sides we were originally given have three significant digits.

We can find *c* using the Pythagorean Theorem or one of our trigonometric functions.

Example 2 – *Solution*

cont'd

Let's start with a trigonometric function.

If $\quad \sin A = \dfrac{a}{c}$

then $\quad c = \dfrac{a}{\sin A} \qquad$ Multiply each side by c, then divide each side by $\sin A$

$$= \frac{2.73}{\sin 38.7°}$$

$$= \frac{2.73}{0.6252}$$

$= 4.37 \qquad$ To three significant digits

Example 2 – *Solution*

cont'd

Using the Pythagorean Theorem, we obtain the same result.

$$\text{If} \qquad c^2 = a^2 + b^2$$

$$\text{then} \qquad c = \sqrt{a^2 + b^2}$$

$$= \sqrt{(2.73)^2 + (3.41)^2}$$

$$= \sqrt{19.081}$$

$$= 4.37$$

Example 5

The first Ferris wheel was designed and built by American engineer George W. G. Ferris in 1893. The diameter of this wheel was 250 feet. It had 36 cars, each of which held 40 passengers. The top of the wheel was 264 feet above the ground. It took 20 minutes to complete one revolution.

Figure 5 is a simplified model of that Ferris wheel. If θ is the central angle formed as a rider moves from position P_0 to position P_1, find the rider's height above the ground h when θ is 45°.

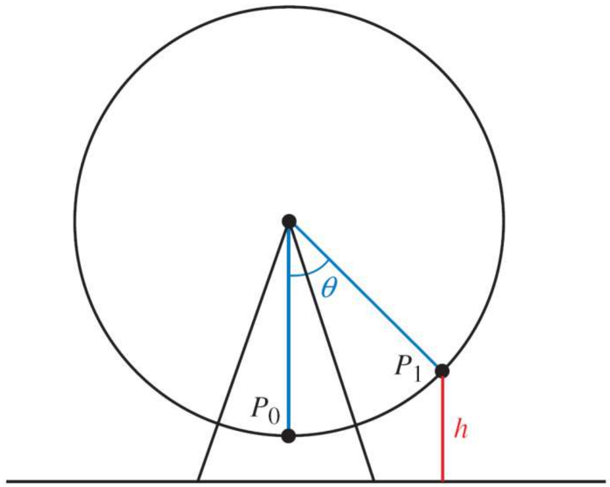

Figure 5

Example 5 – *Solution*

The diameter of the first Ferris wheel was 250 feet, which means the radius was 125 feet.

Because the top of the wheel was 264 feet above the ground, the distance from the ground to the bottom of the wheel was 14 feet (the distance to the top minus the diameter of the wheel).

To form a right triangle, we draw a horizontal line from P_1 to the vertical line connecting the center of the wheel O with P_0.

Example 5 – *Solution*

cont'd

This information is shown in Figure 6.

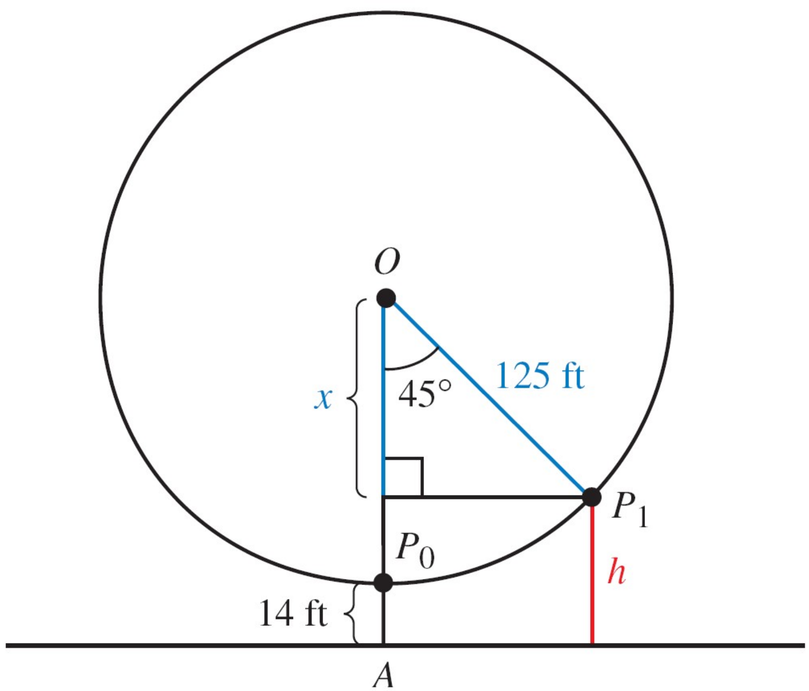

Figure 6

Example 5 – *Solution*

cont'd

The key to solving this problem is recognizing that x is the difference between *OA* (the distance from the center of the wheel to the ground) and *h*.

Because *OA* is 139 feet (the radius of the wheel plus the distance between the bottom of the wheel and the ground: 125 + 14 = 139), we have

$$x = 139 - h$$

We use a cosine ratio to write an equation that contains *h.*

$$\cos 45^\circ = \frac{x}{125}$$

Example 5 – *Solution*

cont'd

$$= \frac{139 - h}{125}$$

Solving for *h* we have

$$125 \cos 45° = 139 - h$$

$$h = 139 - 125 \cos 45°$$

$$= 139 - 125(0.7071)$$

Example 5 – *Solution*

cont'd

$$= 139 - 88.4$$

$$= 51 \text{ ft} \qquad \text{To two significant digits}$$

If $\theta = 45°$, a rider at position P_1 is $\frac{1}{8}$ of the way around the wheel. At that point, the rider is approximately 51 feet above the ground.