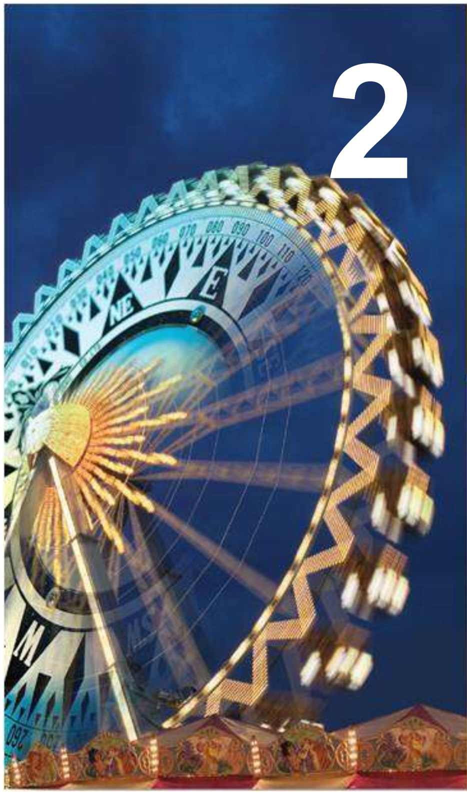

2 Right Triangle Trigonometry

SECTION 2.1

Definition II: Right Triangle Trigonometry

Learning Objectives

1. Find the value of a trigonometric function for an angle in a right triangle.
2. Use the Cofunction Theorem to find the value of a trigonometric function.
3. Find the exact value of a trigonometric function for a special angle.
4. Use exact values to simplify an expression involving trigonometric functions.

Definition II: Right Triangle Trigonometry

DEFINITION II

If triangle ABC is a right triangle with $C = 90°$ (Figure 1), then the six trigonometric functions for A are defined as follows:

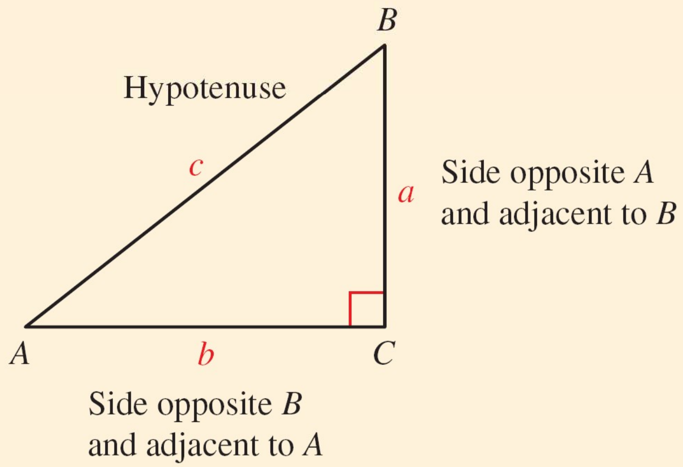

Figure 1

$$\sin A = \frac{\text{side opposite } A}{\text{hypotenuse}} = \frac{a}{c}$$

$$\cos A = \frac{\text{side adjacent } A}{\text{hypotenuse}} = \frac{b}{c}$$

$$\tan A = \frac{\text{side opposite } A}{\text{side adjacent } A} = \frac{a}{b}$$

$$\cot A = \frac{\text{side adjacent } A}{\text{side opposite } A} = \frac{b}{a}$$

$$\sec A = \frac{\text{hypotenuse}}{\text{side adjacent } A} = \frac{c}{b}$$

$$\csc A = \frac{\text{hypotenuse}}{\text{side opposite } A} = \frac{c}{a}$$

Example 1

Triangle *ABC* is a right triangle with $C = 90°$. If $a = 6$ and $c = 10$, find the six trigonometric functions of *A*.

Solution:

We begin by making a diagram of *ABC* (Figure 2) and then use the given information and the Pythagorean Theorem to solve for *b*.

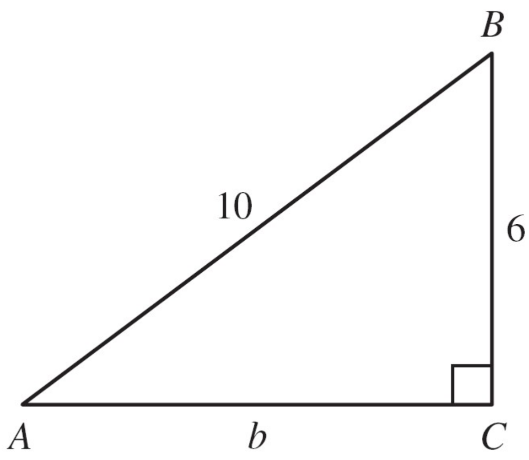

Figure 2

Example 1 – *Solution*

cont'd

$$b = \sqrt{c^2 - a^2}$$

$$= \sqrt{100 - 36}$$

$$= \sqrt{64}$$

$$= 8$$

Now we write the six trigonometric functions of *A* using *a* = 6, *b* = 8, and *c* = 10.

$$\sin A = \frac{a}{c} = \frac{6}{10} = \frac{3}{5} \qquad \csc A = \frac{c}{a} = \frac{5}{3}$$

Example 1 – *Solution*

cont'd

$$\cos A = \frac{b}{c} = \frac{8}{10} = \frac{4}{5} \qquad \sec A = \frac{c}{b} = \frac{5}{4}$$

$$\tan A = \frac{a}{b} = \frac{6}{8} = \frac{3}{4} \qquad \cot A = \frac{b}{a} = \frac{4}{3}$$

Definition II: Right Triangle Trigonometry

DEFINITION

Sine and *co*sine are *co*functions, as are tangent and *co*tangent, and secant and *co*secant. We say sine is the cofunction of cosine, and cosine is the cofunction of sine.

Now let's see what happens when we apply Definition II to *B* in right triangle *ABC*.

$$\sin B = \frac{\text{side opposite } B}{\text{hypotenuse}} = \frac{b}{c} = \cos A$$

Definition II: Right Triangle Trigonometry

$$\cos B = \frac{\text{side adjacent } B}{\text{hypotenuse}} = \frac{a}{c} = \sin A$$

$$\tan B = \frac{\text{side opposite } B}{\text{side adjacent } B} = \frac{b}{a} = \cot A$$

$$\cot B = \frac{\text{side adjacent } B}{\text{side opposite } B} = \frac{a}{b} = \tan A$$

$$\sec B = \frac{\text{hypotenuse}}{\text{side adjacent } B} = \frac{c}{a} = \csc A$$

$$\csc B = \frac{\text{hypotenuse}}{\text{side opposite } B} = \frac{c}{b} = \sec A$$

Definition II: Right Triangle Trigonometry

As you can see in Figure 4, every trigonometric function of *A* is equal to the cofunction of *B*.

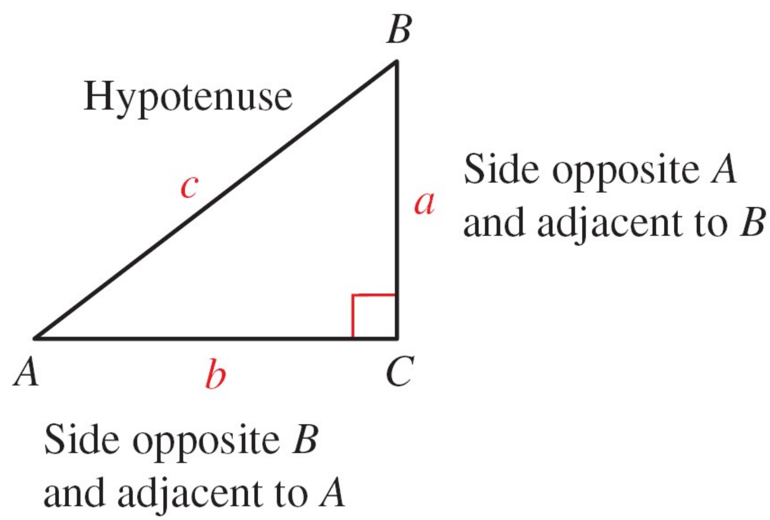

Figure 4

Definition II: Right Triangle Trigonometry

That is, sin A = cos B, sec A = csc B, and tan A = cot B, to name a few.

Because A and B are the acute angles in a right triangle, they are always complementary angles; that is, their sum is always 90°.

What we actually have here is another property of trigonometric functions:

The sine of an angle is the cosine of its complement, the secant of an angle is the cosecant of its complement, and the tangent of an angle is the cotangent of its complement.

Definition II: Right Triangle Trigonometry

Or, in symbols,

$$\text{if } A + B = 90^\circ \text{, then} \quad \begin{cases} \sin A = \cos B \\ \sec A = \csc B \\ \tan A = \cot B \end{cases}$$

and so on.

We generalize this discussion with the following theorem.

COFUNCTION ■ THEOREM

A trigonometric function of an angle is always equal to the cofunction of the complement of the angle.

Definition II: Right Triangle Trigonometry

To clarify this further, if two angles are complementary, such as 40° and 50°, then a trigonometric function of one is equal to the cofunction of the other.

That is, sin 40° = cos 50°, sec 40° = csc 50°, and tan 40° = cot 50°.

Example 3

Fill in the blanks so that each expression becomes a true statement.

a. $\sin \_\_\_\_\_ = \cos 30°$ **b.** $\tan y = \cot \_\_\_\_\_$

c. $\sec 75° = \csc \_\_\_\_\_$

Solution:

Using the theorem on cofunctions of complementary angles, we fill in the blanks as follows:

a. $\sin \underline{\quad 60° \quad} = \cos 30°$ Because sine and cosine are cofunctions and $60° + 30° = 90°$

Example 3 – *Solution*

cont'd

b. $\tan y = \cot \underline{(90° - y)}$

Because tangent and cotangent are cofunctions and $y + (90° - y) = 90°$

c. $\sec 75° = \csc \underline{15°}$

Because secant and cosecant are cofunctions and $75° + 15° = 90°$

Definition II: Right Triangle Trigonometry

For our next application of Definition II, let us consider the two special triangles 30°–60°–90° triangle and the 45°–45°–90° triangle. Figure 5 shows both of these triangles for the case in which the shortest side is 1 (t = 1).

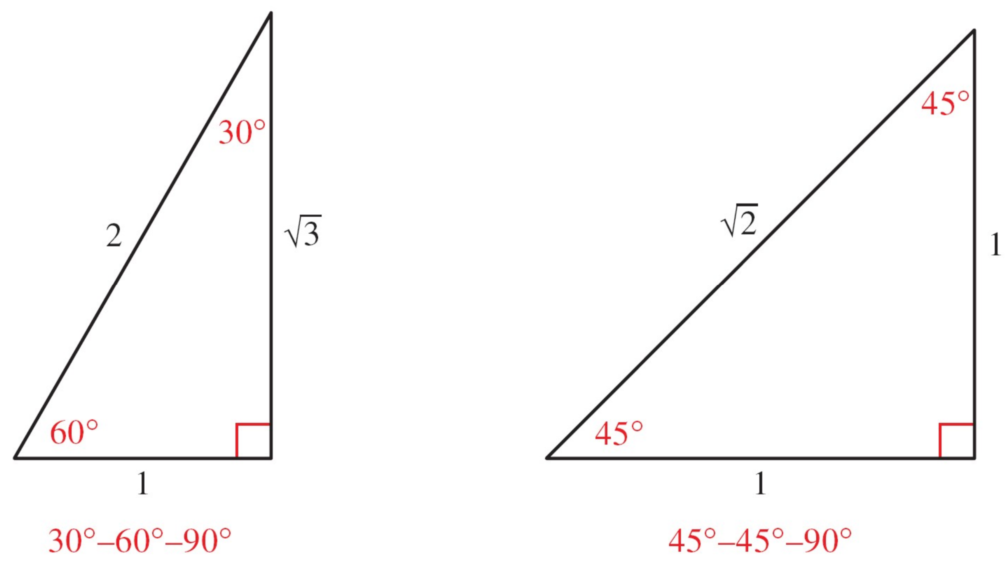

Figure 5

Definition II: Right Triangle Trigonometry

Using these two special triangles and Definition II, we can find the trigonometric functions of 30°, 45°, and 60°.

If we were to continue finding the sine, cosine, and tangent for these special angles (Figure 6), we would obtain the results summarized in Table 1.

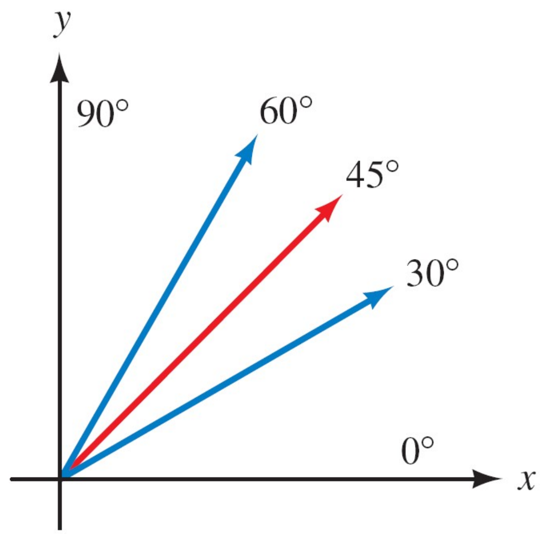

Figure 6

Definition II: Right Triangle Trigonometry

Because we will be using these values so frequently, you should either memorize the special triangles in Figure 5 or the information in Table 1.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
45°	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

Exact Values

Table 1

Table 1 is called a table of exact values to distinguish it from a table of approximate values.

Example 5

Let $x = 30°$ and $y = 45°$ in each of the expressions that follow, and then simplify each expression as much as possible.

a. $2 \sin x$ **b.** $\sin 2y$ **c.** $4 \sin (3x - 90°)$

Solution:

a. $2 \sin x = 2 \sin 30° = 2(1/2) = 1$

b. $\sin 2y = \sin 2(45°) = \sin 90° = 1$

c. $4 \sin (3x - 90°) = 4 \sin [3(30°) - 90°] = 4 \sin 0° = 4(0) = 0$

Definition II: Right Triangle Trigonometry

To conclude this section, we take the information previously obtained for 0° and 90°, along with the exact values in Table 1, and summarize them in Table 2.

θ	**0°**	**30°**	**45°**	**60°**	**90°**
$\sin\theta$	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
$\cos\theta$	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$
$\tan\theta$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

Table 2

Definition II: Right Triangle Trigonometry

To make the information in Table 2 a little easier to memorize, we have written some of the exact values differently than we usually do. For example, in Table 2 we have written 2 as $\sqrt{4}$, 0 as $\sqrt{0}$, and 1 as $\sqrt{1}$.