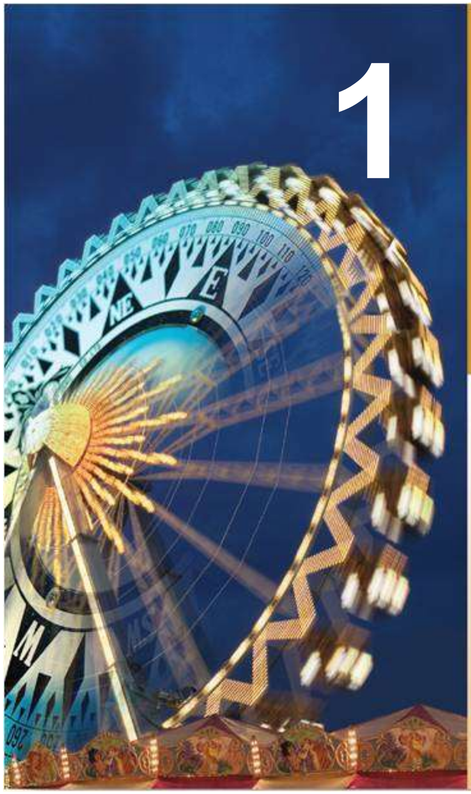

1 The Six Trigonometric Functions

SECTION 1.3

Definition I: Trigonometric Functions

Objectives

1. Find the value of a trigonometric function of an angle given a point on the terminal side.
2. Use Definition I to answer a conceptual question about a trigonometric function.
3. Determine the quadrants an angle could terminate in.
4. Find the value of a trigonometric function given one of the other values.

Definition I: Trigonometric Functions

DEFINITION I

If θ is an angle in standard position, and the point (x, y) is any point on the terminal side of θ other than the origin, then the six trigonometric functions of angle θ are defined as follows:

Function		Abbreviation		Definition
The sine of θ	=	$\sin\theta$	=	$\frac{y}{r}$
The cosine of θ	=	$\cos\theta$	=	$\frac{x}{r}$
The tangent of θ	=	$\tan\theta$	=	$\frac{y}{x}$ $(x \neq 0)$
The cotangent of θ	=	$\cot\theta$	=	$\frac{x}{y}$ $(y \neq 0)$
The secant of θ	=	$\sec\theta$	=	$\frac{r}{x}$ $(x \neq 0)$
The cosecant of θ	=	$\csc\theta$	=	$\frac{r}{y}$ $(y \neq 0)$

where $x^2 + y^2 = r^2$, or $r = \sqrt{x^2 + y^2}$. That is, r is the distance from the origin to (x, y).

Definition I: Trigonometric Functions

As you can see, the six trigonometric functions are simply names given to the six possible ratios that can be made from the numbers *x*, *y*, and *r* as shown in Figure 1.

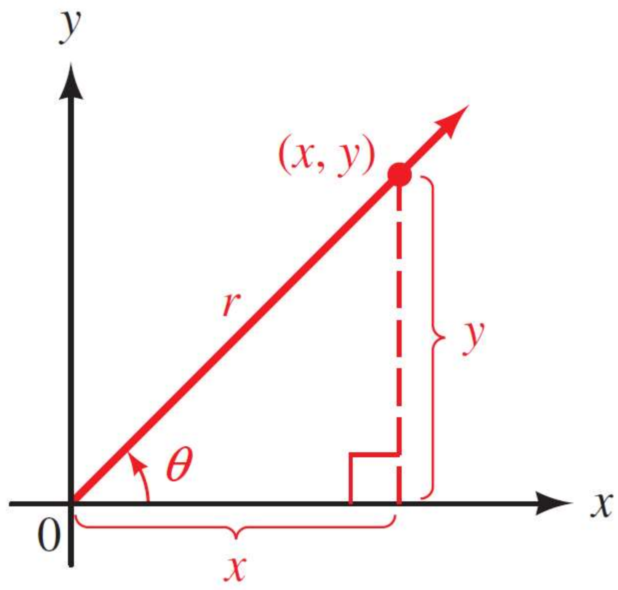

Figure 1

Definition I: Trigonometric Functions

In particular, notice that tan θ can be interpreted as the slope of the line corresponding to the terminal side of θ.

Both tan θ and sec θ will be undefined when $x = 0$, which will occur any time the terminal side of θ coincides with the *y*-axis.

Likewise, both cot θ and csc θ will be undefined when $y = 0$, which will occur any time the terminal side of θ coincides with the *x*-axis.

Example 1

Find the six trigonometric functions of θ if θ is in standard position and the point (–2, 3) is on the terminal side of θ.

Solution:

We begin by making a diagram showing θ, (–2, 3), and the distance r from the origin to (–2, 3), as shown in Figure 2.

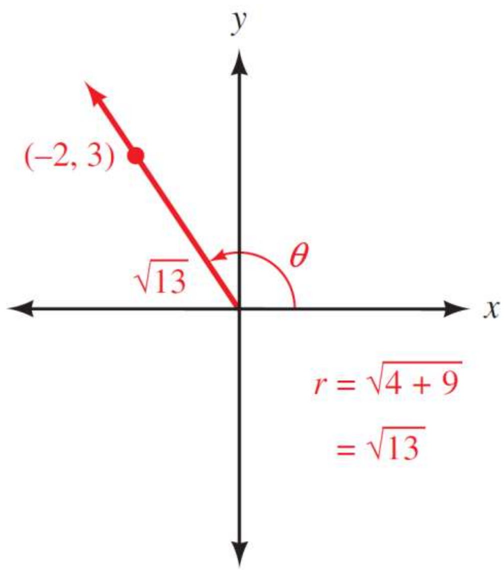

Figure 2

Example 1 – *Solution*

cont'd

Applying the definition for the six trigonometric functions using the values $x = -2$, $y = 3$, and $r = \sqrt{13}$, we have

$$\sin\theta = \frac{y}{r} = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13} \qquad \csc\theta = \frac{r}{y} = \frac{\sqrt{13}}{3}$$

$$\cos\theta = \frac{x}{r} = -\frac{2}{\sqrt{13}} = -\frac{2\sqrt{13}}{13} \qquad \sec\theta = \frac{r}{x} = -\frac{\sqrt{13}}{2}$$

$$\tan\theta = \frac{y}{x} = -\frac{3}{2} \qquad \cot\theta = \frac{x}{y} = -\frac{2}{3}$$

Definition I: Trigonometric Functions

Note: In algebra, when we encounter expressions like $3/\sqrt{13}$ that contain a radical in the denominator, we usually rationalize the denominator; in this case, by multiplying the numerator and the denominator by $\sqrt{13}$

$$\frac{3}{\sqrt{13}} = \frac{3}{\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$$

In trigonometry, it is sometimes convenient to use $3\sqrt{13}/13$, and at other times it is easier to use $3/\sqrt{13}$.

In most cases we will go ahead and rationalize denominators, but you should check and see if your instructor has a preference either way.

Algebraic Signs of Trigonometric Functions

Algebraic Signs of Trigonometric Functions

The algebraic sign, + or –, of each of the six trigonometric functions will depend on the quadrant in which θ terminates.

For example, in quadrant I all six trigonometric functions are positive because *x*, *y*, and *r* are all positive.

In quadrant II, only sin θ and csc θ are positive because *y* and *r* are positive and *x* is negative.

Algebraic Signs of Trigonometric Functions

Table 1 shows the signs of all the ratios in each of the four quadrants.

TABLE 1

For θ in	**QI**	**QII**	**QIII**	**QIV**
$\sin\theta = \frac{y}{r}$ and $\csc\theta = \frac{r}{y}$	+	+	−	−
$\cos\theta = \frac{x}{r}$ and $\sec\theta = \frac{r}{x}$	+	−	−	+
$\tan\theta = \frac{y}{x}$ and $\cot\theta = \frac{x}{y}$	+	−	+	−

Example 5

If sin $\theta = -5/13$, and θ terminates in quadrant III, find cos θ and tan θ.

Solution:

Because sin $\theta = -5/13$, we know the ratio of y to r, or y/r, is $-5/13$. We can let y be -5 and r be 13 and use these values of y and r to find x.

Figure 6 shows θ in standard position with the point on the terminal side of θ having a y-coordinate of -5.

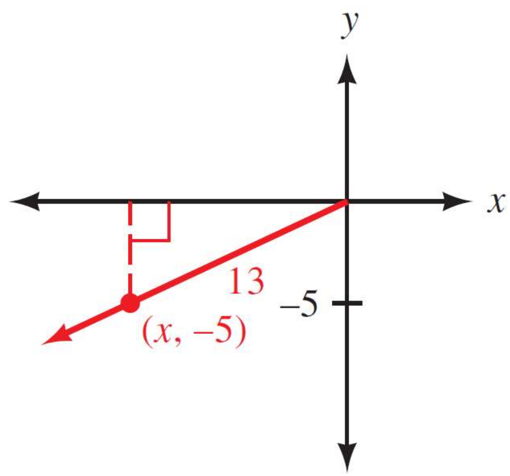

Figure 6

Example 5 – *Solution*

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To find *x*, we use the fact that $x^2 + y^2 = r^2$.

$$x^2 + y^2 = r^2$$

$$x^2 + (-5)^2 = 13^2$$

$$x^2 + 25 = 169$$

$$x^2 = 144$$

$$x = \pm 12$$

Example 5 – *Solution*

cont'd

Is *x* the number 12 or –12?

Because θ terminates in quadrant III, we know any point on its terminal side will have a negative *x*-coordinate; therefore,

$$x = -12$$

Using *x* = –12, *y* = –5, and *r* = 13 in our original definition, we have

$$\cos\theta = \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13} \quad \text{and} \quad \tan\theta = \frac{y}{x} = \frac{-5}{-12} = \frac{5}{12}$$

Algebraic Signs of Trigonometric Functions

As a final note, we should emphasize that the trigonometric functions of an angle are independent of the choice of the point (x, y) on the terminal side of the angle.

Figure 7 shows an angle θ in standard position.

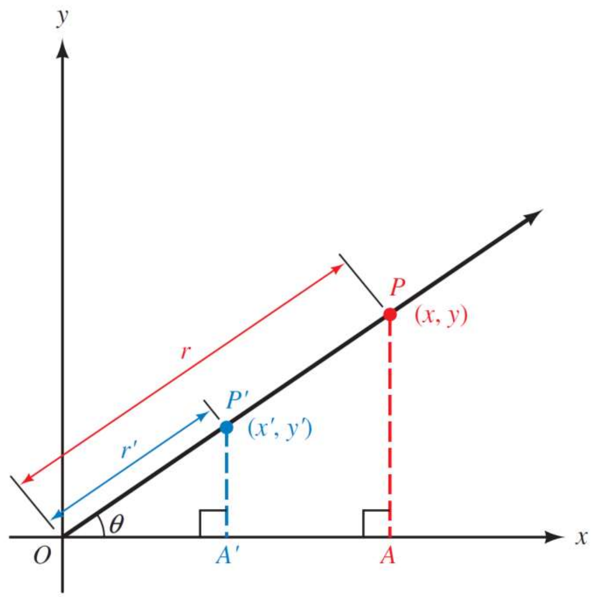

Figure 7

Algebraic Signs of Trigonometric Functions

Points $P(x, y)$ and $P'(x', y')$ are both points on the terminal side of θ.

Because triangles $P'OA'$ and POA are similar triangles, their corresponding sides are proportional.

That is,

$$\sin \theta = \frac{y'}{r'} = \frac{y}{r} \qquad \cos \theta = \frac{x'}{r'} = \frac{x}{r} \qquad \tan \theta = \frac{y'}{x'} = \frac{y}{x}$$