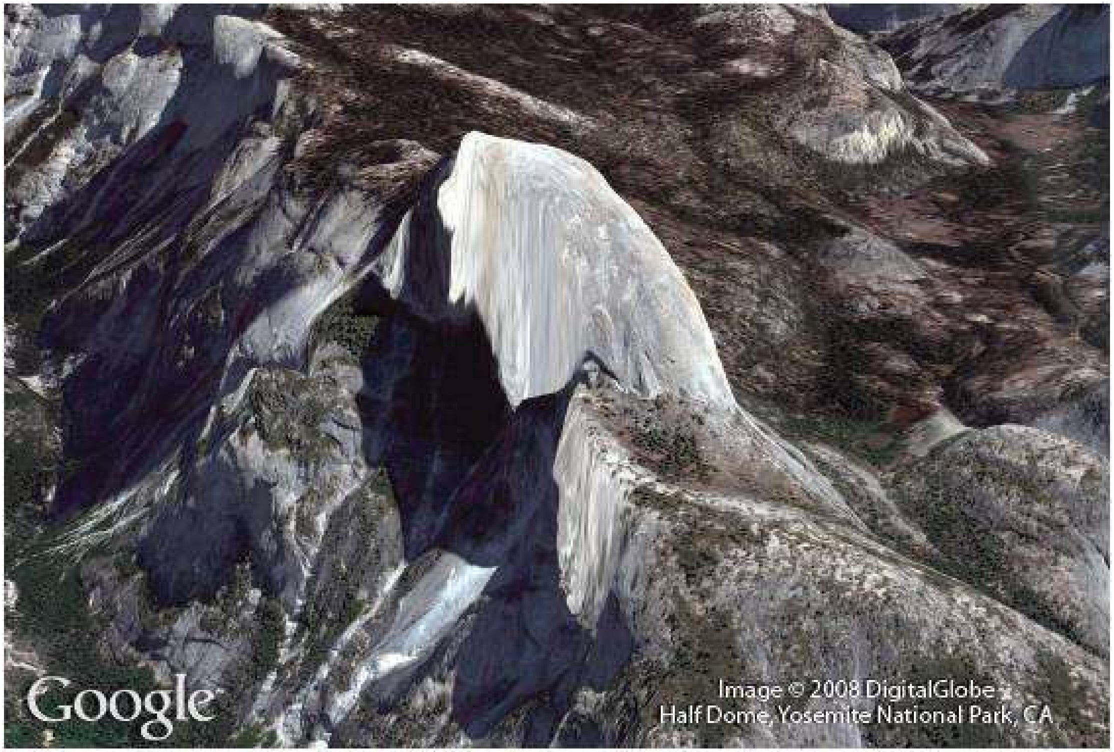
# Exponents and Polynomials

# 9



SECTION 9.3

# Multiplying Polynomials: An Introduction

# Objectives

**A** Multiply polynomials.

**B** Multiply binomials.

## A Multiplying Polynomials Algebraically

# Multiplying Polynomials Algebraically

The distributive property allows us to multiply across parentheses when a sum or difference is enclosed within the parentheses.

That is,

$$a(b + c) = a \cdot b + a \cdot c$$

We can use the distributive property to multiply polynomials.

# Multiplying Polynomials Algebraically

The distributive property works for multiplication from the right as well as the left.

That is, we can also write the distributive property this way:

$$(b + c)a = b \cdot a + c \cdot a$$

# Example 4

Multiply: $2a^4b^2(3a^3 + 4b^5)$.

Solution:

Applying the distributive property, we have

$$\mathbf{2a^4b^2}(3a^3 + 4b^5) = \mathbf{2a^4b^2} \cdot 3a^3 + \mathbf{2a^4b^2} \cdot 4b^5$$

$$= (2 \cdot 3)(a^4 \cdot a^3)b^2 + (2 \cdot 4)(a^4)(b^2 \cdot b^5)$$

$$= 6a^7b^2 + 8a^4b^7$$

## B Multiplying Binomials

# Multiplying Binomials

Polynomials with exactly two terms are called *binomials.*
We multiply binomials by applying the distributive property.

# Example 5

Multiply: $(x + 3)(x + 5)$.

Solution:

We can think of the first binomial, *x* + 3, as a single number. (Remember, for any value of *x*, *x* + 3 will be just a number.)

We apply the distributive property by multiplying *x* + 3 times both *x* and 5.

$$(x + 3)(x + 5) = \mathbf{(x + 3)} \cdot x + \mathbf{(x + 3)} \cdot 5$$

# Example 5 – *Solution*

cont'd

Next, we apply the distributive property again to multiply $x$ times both $x$ and 3, and 5 times both $x$ and 3.

$$= x \cdot \mathbf{x} + 3 \cdot \mathbf{x} + x \cdot \mathbf{5} + 3 \cdot \mathbf{5}$$

$$= x^2 + 3x + 5x + 15$$

The last thing to do is to combine the similar terms $3x$ and $5x$ to get $8x$. (Remember, this is also an application of the distributive property.)

$$= x^2 + 8x + 15$$

# Multiplying Binomials

If we look closely at the second and third lines of work in the previous example, we can see that the terms in the answer come from all possible products of terms in the first binomial with terms in the second binomial.

This result is generalized as follows.

**Rule Multiplying Two Polynomials**

To multiply any two polynomials, multiply each term in the first with each term in the second.

# FOIL Method

# FOIL Method

If we look at the problem in Example 5 and then at the answer, we see that the first term in the answer came from multiplying the first terms in each binomial.

$$x \cdot x = x^2 \qquad \text{First}$$

The middle term in the answer came from adding the product of the two outside terms in each binomial and the product of the two inside terms in each binomial.

$$x(5) = 5x \qquad \text{Outside}$$

# FOIL Method

$$3(x) = 3x \qquad \text{Inside}$$

$$5x + 3x = 8x$$

The last term in the answer came from multiplying the two last terms:

$$3(5) = 15 \qquad \text{Last}$$

To summarize the FOIL method, we will multiply another two binomials.

# Example 6

Multiply: $(2x + 3)(5x - 4)$.

Solution:

$$(2x + 3)(5x - 4) = \underbrace{2x(5x)}_{\text{First}} + \underbrace{2x(-4)}_{\text{Outside}} + \underbrace{3(5x)}_{\text{Inside}} + \underbrace{3(-4)}_{\text{Last}}$$

$$= 10x^2 - 8x + 15x - 12$$

# Example 6 – *Solution*

cont'd

$$= 10x^2 + 7x - 12$$

With practice, adding the products of the outside and inside terms, $-8x + 15x = 7x$ can be done mentally.

# Example 9

Expand and multiply: $(3x - 7)^2$.

Solution:

We know that $(3x - 7)^2 = (3x - 7)(3x - 7)$.

It will be easier to apply the distributive property to this last expression if we think of the second $3x - 7$ as $3x + (-7)$.

In doing so we will also be less likely to make a mistake in our signs. (Try the problem without changing subtraction to addition of the opposite, and see how your answer compares to the answer in this example.)

# Example 9 – *Solution*

cont'd

$$(3x - 7)(3x - 7) = (3x - 7)[3x + (-7)]$$

$$= (3x - 7) \cdot 3x + (3x - 7)(-7)$$

$$= 3x \cdot 3x - 7 \cdot 3x + 3x(-7) - 7(-7)$$

$$= 9x^2 - 21x - 21x + 49$$

$$= 9x^2 - 42x + 49$$

# Column Method

# Column Method

The FOIL method can be applied only when multiplying two binomials.

To find products of polynomials with more than two terms, we use what is called the Column method.

The Column method of multiplying two polynomials is very similar to long multiplication with whole numbers.

It is just another way of finding all possible products of terms in one polynomial with terms in another polynomial.

# Example 10

Multiply $(2x + 3)(3x^2 - 2x + 1)$.

Solution:

$$
\begin{array}{rl}
3x^2 - 2x + 1 & \\
2x + 3 & \\
\hline
6x^3 - 4x^2 + 2x \phantom{{}+3} & \leftarrow \quad 2x\,(3x^2 - 2x + 1) \\
9x^2 - 6x + 3 & \leftarrow \quad 3(3x^2 - 2x + 1) \\
\hline
6x^3 + 5x^2 - 4x + 3 & \quad \text{Add similar terms.}
\end{array}
$$