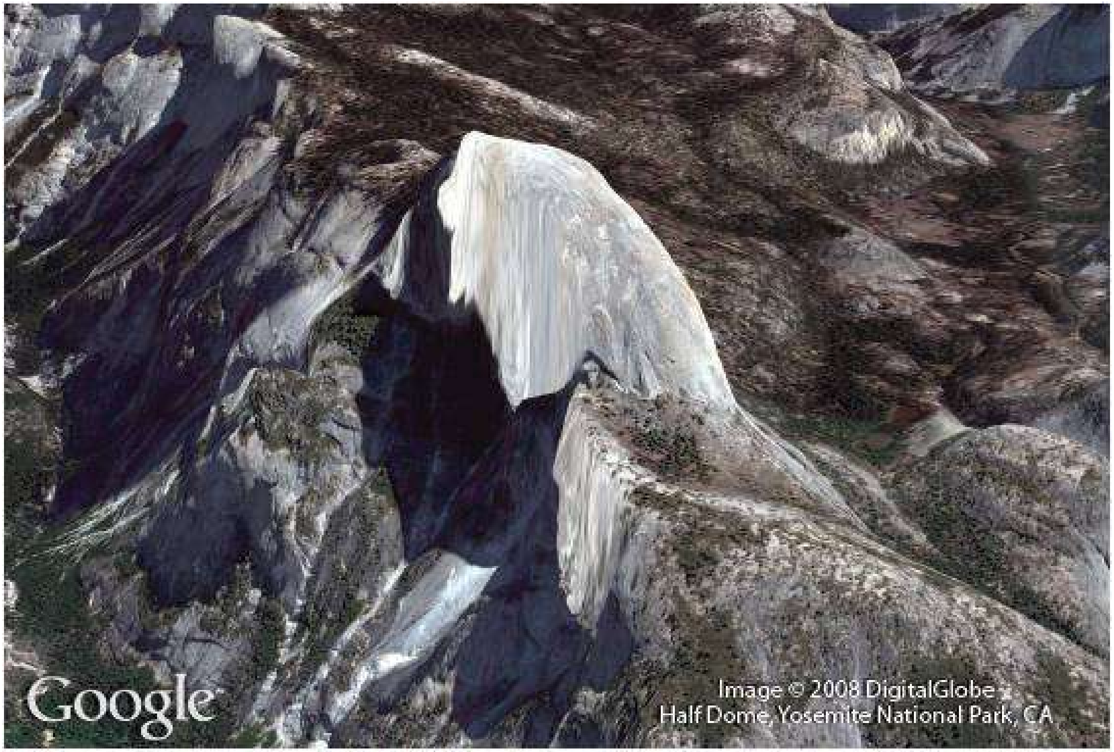
Exponents and Polynomials

9



SECTION 9.2

Adding and Subtracting Polynomials

Objectives

A Add polynomials.

B Subtract polynomials.

C Find the value of a polynomial.

Adding and Subtracting Polynomials

We have written the numbers in expanded form to show the place value of each of the digits. For example, the number 456 in expanded form looks like this:

$$456 = 4 \cdot 100 + 5 \cdot 10 + 6 \cdot 1$$

If we replace 100 with 10^2, we have

$$456 = 4 \cdot 10^2 + 5 \cdot 10 + 6 \cdot 1$$

If we replace the 10′s with x′s, we get an expression with numbers and variables called a *polynomial*.

Adding and Subtracting Polynomials

It looks like this:

$$4x^2 + 5x + 6$$

Polynomials are to algebra as whole numbers written in expanded form are to arithmetic.

As in other expressions in algebra, we can use any variable we choose. The following expressions are also polynomials:

$$5a + 3 \qquad y^2 + 2y + 4 \qquad x^3 - 2x^2 + 5x - 1$$

A Adding Polynomials

Adding Polynomials

When we add two whole numbers, we add in columns. That is, if we add 234 and 345, we write one number under the other and add the numbers in the ones column, then the numbers in the tens column, and finally the numbers in the hundreds column.

Here is how it looks:

$$\begin{array}{r} 234 \\ +\ 345 \\ \hline 579 \end{array} \qquad \begin{array}{r} \overbrace{2 \cdot 10^2}^{\text{Hundreds}} + \overbrace{3 \cdot 10}^{\text{Tens}} + \overbrace{4}^{\text{Ones}} \\ +\ 3 \cdot 10^2 + 4 \cdot 10 + 5 \\ \hline 5 \cdot 10^2 + 7 \cdot 10 + 9 \end{array}$$

Adding Polynomials

We add polynomials in the same manner. If we want to add $2x^2 + 3x + 4$ and $3x^2 + 4x + 5$, we write one polynomial under the other, and then add in columns:

$$\begin{array}{r} 2x^2 + 3x + 4 \\ +\ 3x^2 + 4x + 5 \\ \hline 5x^2 + 7x + 9 \end{array}$$

The sum of the two polynomials is the polynomial $5x^2 + 7x + 9$. We add only the digits.

Notice that the variable parts (the x's) stay the same, just as the powers of 10 did when we added 234 and 345.

Adding Polynomials

The reason we add the numbers, while the variable parts of each term stay the same, can be explained with the commutative, associative, and distributive properties.

Example 2

Add $6y^2 + 4y - 3$ and $2y^2 - 4$.

Solution:

We write one polynomial under the other, so that the terms with y^2 line up, the terms with the y's line up, and the terms without any y's line up.

$$\begin{array}{r} 6y^2 + 4y - 3 \\ +\ 2y^2 - 4 \\ \hline 8y^2 + 4y - 7 \end{array}$$

The same problem, written horizontally, looks like this:

$$(6y^2 + 4y - 3) + (2y^2 - 4) = (6y^2 + 2y^2) + 4y + (-3 - 4)$$

Example 2 – *Solution*

cont'd

$$= (6 + 2)y^2 + 4y + (-3 - 4)$$

$$= 8y^2 + 4y + (-7)$$

$$= 8y^2 + 4y - 7$$

B Subtracting Polynomials

Subtracting Polynomials

If there is a negative sign directly preceding the parentheses surrounding a polynomial, we may remove the parentheses and preceding negative sign by applying the distributive property.

Example 4

Subtract: $(6x^2 - 3x + 5) - (3x^2 + 2x - 3)$.

Solution:

Because subtraction is addition of the opposite, we simply change the sign of each term of the second polynomial and then add.

With subtraction, there is less chance of making mistakes if we add horizontally, rather than in columns.

$$(6x^2 - 3x + 5) - (3x^2 + 2x - 3) \qquad \text{Subtract.}$$

$$= 6x^2 - 3x + 5 - 3x^2 - 2x + 3 \qquad \text{Add the opposite.}$$

Example 4 – *Solution*

cont'd

$$= (6x^2 - 3x^2) + (-3x - 2x) + (5 + 3)$$

$$= (6 - 3)x^2 + (-3 - 2)x + (5 + 3)$$ Distributive property

$$= 3x^2 - 5x + 8$$

C Finding the Value of a Polynomial

Finding the Value of a Polynomial

We can find the values of algebraic expressions in earlier sections. Now we will apply the same process to a polynomial.

Example 7

Find the value of $4x^2 - 7x + 2$ when $x = -2$.

Solution:

When $x = -2$, the polynomial $4x^2 - 7x + 2$ becomes

$$
\begin{aligned}
4(-2)^2 - 7(-2) + 2 &= 4(4) + 14 + 2 \\
&= 16 + 14 + 2 \\
&= 32
\end{aligned}
$$