Exponents and Polynomials

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SECTION 9.1

Multiplication Properties of Exponents

Objectives

A   Understand and apply the multiplication property for exponents.

B   Understand and apply the power property for exponents.

C   Understand and apply the product property for exponents.

D   Simplify expressions using two or more properties of exponents.

A **Multiplication Property for Exponents**

Multiplication Property for Exponents

Let's begin by looking at how we multiply two exponential expressions.

Example 1

Multiply: $x^2 \cdot x^4$.

Solution:

We write x^2 as $x \cdot x$ and x^4 as $x \cdot x \cdot x \cdot x$ and then use the definition of exponents to write the answer with just one exponent.

$$x^2 \cdot x^4 = (x \cdot x)(x \cdot x \cdot x \cdot x)$$ Expand exponents.

$$= x \cdot x \cdot x \cdot x \cdot x \cdot x$$

$$= x^6$$ Write with a single exponent.

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Multiplication Property for Exponents

Notice that the two exponents in the original problem given in Example 1 add up to the exponent in the answer: 2 + 4 = 6.

We use this result as justification for writing our first property of exponents.

Multiplication Property for Exponents

If r and s are any two whole numbers and a is an integer, then

$$a^r \cdot a^s = a^{r+s}$$

In words: To multiply two expressions with the same base, add exponents and use the common base.

Example 4

Multiply: $(2x^2y^3)(7x^4y)$.

Solution:

This time we have two different variables to work with. We use the commutative and associative properties to group the numbers together, the x's together, and the y's together.

$$(2x^2y^3)(7x^4y) = (2 \cdot 7)(x^2 \cdot x^4)(y^3 \cdot y)$$

$$= (2 \cdot 7)(x^{2+4})(y^{3+1})$$

$$= 14x^6y^4$$

B Power Property for Exponents

Power Property for Exponents

Another common expression in algebra is that of a power raised to another power— for example, $(x^2)^3$, $(10^4)^5$, and $(y^6)^7$.

To see how these expressions can be written with a single exponent, we return to the definition of exponents.

$$(x^2)^3 = x^2 \cdot x^2 \cdot x^2 \qquad \text{Definition of exponents}$$
$$= x^{2+2+2} \qquad \text{Multiplication property for exponents}$$
$$= x^6 \qquad \text{Add exponents.}$$

Power Property for Exponents

As you can see, to obtain the exponent in the answer, we multiply the exponents in the original problem.

The result leads us to our second property of exponents.

Power Property for Exponents

If r and s are any two whole numbers and a is an integer, then

$$(a^r)^s = a^{r \cdot s}$$

In words: A power raised to another power is the base raised to the product of the powers.

Example 6

Simplify: $(x^5)^6 \cdot (x^4)^7$.

Solution:

In this case, we apply the power property for exponents first to write each expression with a single exponent. Then we apply the multiplication property to simplify further.

$$(x^5)^6 \cdot (x^4)^7 = x^{5 \cdot 6} \cdot x^{4 \cdot 7}$$ Power property for exponents

$$= x^{30} \cdot x^{28}$$ Multiply exponents.

$$= x^{30+28}$$ Multiplication property for exponents

$$= x^{58}$$ Add exponents.

c Product Property for Exponents

Product Property for Exponents

The third, and final, property of exponents for this section covers the situation that occurs when we have the product of more than one number or variable, all raised to a power.

Expressions such as $(4x)^2$, $(2ab)^3$, and $(5x^2y^3)^5$ all fall into this category.

$$(4x)^2 = (4x)(4x)$$
Definition of exponents

$$= (4 \cdot 4)(x \cdot x)$$
Commutative and associative properties

$$= 4^2 \cdot x^2$$
Definition of exponents

Product Property for Exponents

As you can see, both the numbers, 4 and x, contained within the parentheses end up raised to the second power.

Product Property for Exponents

If r is a whole number and a and b are integers, then

$$(a \cdot b)^r = a^r \cdot b^r$$

Example 7

Simplify: $(3xy)^4$.

Solution:

Applying our new property, we have

$$(3xy)^4 = 3^4 \cdot x^4 \cdot y^4 \qquad \text{Product property for exponents}$$

$$= 81x^4y^4 \qquad \text{Simplify.}$$

D Combining Two or More Properties of Exponents

Combining Two or More Properties of Exponents

The next example shows how we can combine two or more properties of exponents to simplify expressions.

Example 9

Simplify: $(4x^3y)^2(5x^2y^4)^3$.

Solution:

This simplification will require all three properties of exponents, as well as the commutative and associative properties of multiplication. Here are all the steps:

$$(4x^3y)^2(5x^2y^4)^3 = 4^2(x^3)^2y^2 \cdot 5^3(x^2)^3(y^4)^3 \qquad \text{Product property for exponents}$$

$$= 16x^6y^2 \cdot 125x^6y^{12} \qquad \text{Power property for exponents}$$

$$= (16 \cdot 125)(x^6x^6)(y^2y^{12}) \qquad \text{Commutative and associative properties}$$

$$= 2{,}000x^{12}y^{14} \qquad \text{Power property for exponents}$$