Measurement

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SECTION 8.2

Unit Analysis II: Area and Volume

Objectives

A   Convert between areas using the U.S. system.

B   Convert between areas using the metric system.

C   Convert between volumes using the U.S. system.

D   Convert between volumes using the metric system.

Unit Analysis II: Area and Volume

Figure 1 below gives a summary of the geometric objects along with the formulas for finding the area of each object.
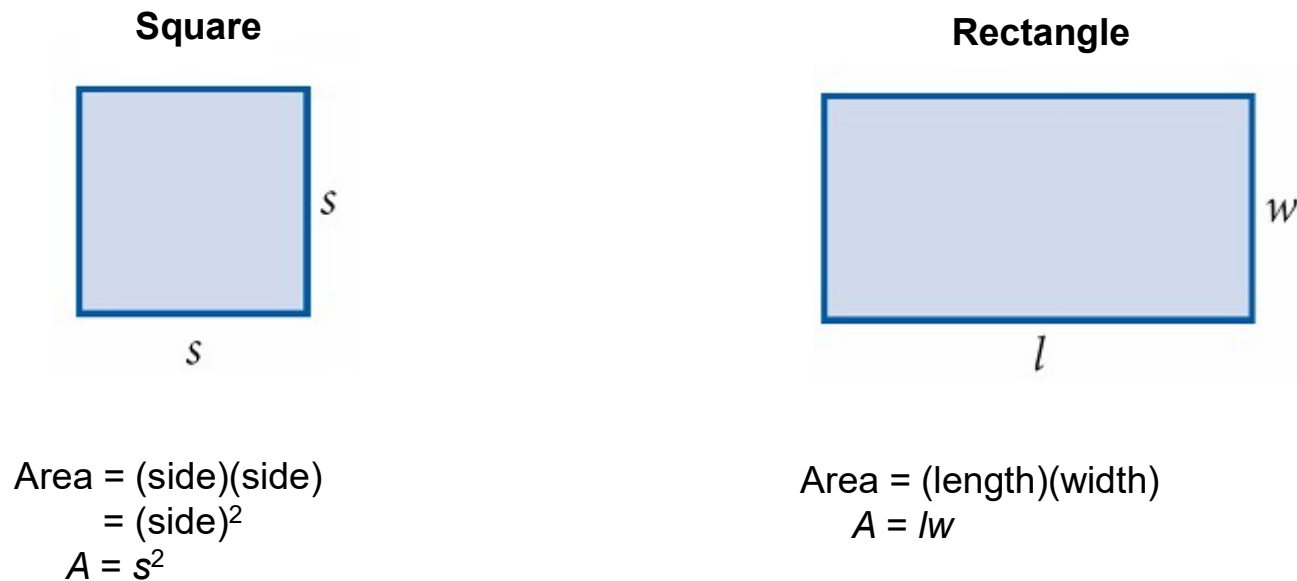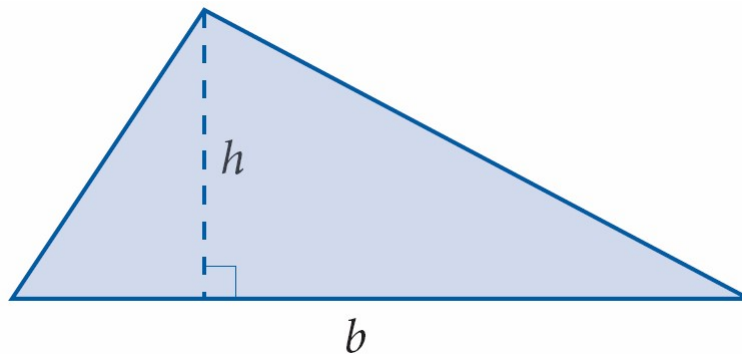
Square

s

s

Area = (side)(side)
= (side)^2
$A = s^2$

Rectangle

w

l

Area = (length)(width)
$A = lw$

Figure 1

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Unit Analysis II: Area and Volume

Triangle

Circle

Area = $\frac{1}{2}$(base)(height)

$A = \frac{1}{2}bh$
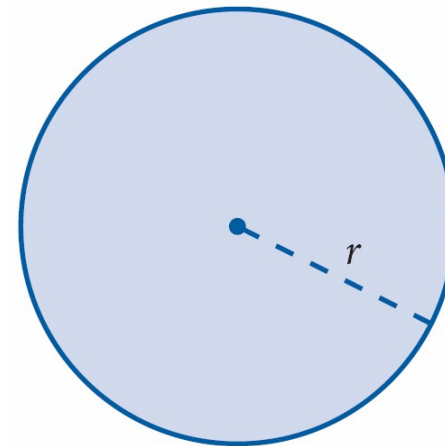
Area = π(radius)^2

$A = \pi r^2$

Figure 1 (continued)

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A Area: The U.S. System

Example 1

Find the number of square inches in 1 square foot.

Solution:

We can think of 1 square foot as $1 \text{ ft}^2 = 1 \text{ ft} \times \text{ft}$.

To convert from feet to inches, we use the conversion factor 1 foot = 12 inches.

Because the unit foot appears twice in 1 ft^2, we multiply by our conversion factor twice.

$$1 \text{ ft}^2 = 1 \text{ ft} \times \text{ft} \times \frac{12 \text{ in.}}{1 \text{ ft}} \times \frac{12 \text{ in.}}{1 \text{ ft}} = 12 \times 12 \text{ in.} \times \text{in.} = 144 \text{ in}^2$$

Area: The U.S. System

Now that we know that 1 ft^2 is the same as 144 in^2, we can use this fact as a conversion factor to convert between square feet and square inches.

Depending on which units we are converting from, we would use either

$$\frac{144 \text{ in}^2}{1 \text{ ft}^2} \quad \text{or} \quad \frac{1 \text{ ft}^2}{144 \text{ in}^2}$$

Area: The U.S. System

Table 1 gives the most common units of area in the U.S. system of measurement, along with the corresponding conversion factors.

TABLE 1

U.S. UNITS OF AREA

The relationship between	is	To convert one to the other, multiply by
square inches and square feet	$144 \text{ in}^2 = 1 \text{ ft}^2$	$\dfrac{144 \text{ in}^2}{1 \text{ ft}^2}$ or $\dfrac{1 \text{ ft}^2}{144 \text{ in}^2}$
square yards and square feet	$9 \text{ ft}^2 = 1 \text{ yd}^2$	$\dfrac{9 \text{ ft}^2}{1 \text{ yd}^2}$ or $\dfrac{1 \text{ yd}^2}{9 \text{ ft}^2}$
acres and square feet	$1 \text{ acre} = 43{,}560 \text{ ft}^2$	$\dfrac{43{,}560 \text{ ft}^2}{1 \text{ acre}}$ or $\dfrac{1 \text{ acre}}{43{,}560 \text{ ft}^2}$
acres and square miles	$640 \text{ acres} = 1 \text{ mi}^2$	$\dfrac{640 \text{ acres}}{1 \text{ mi}^2}$ or $\dfrac{1 \text{ mi}^2}{640 \text{ acres}}$

B **Area: The Metric System**

Area: The Metric System

Units of area in the metric system are considerably simpler than those in the U.S. system because metric units are given in terms of powers of 10.

Table 2 lists the conversion factors that are most commonly used.

TABLE 2	METRIC UNITS OF AREA	
The relationship between	is	To convert one to the other, multiply by
square millimeters and square centimeters	$1\ cm^2 = 100\ mm^2$	$\dfrac{100\ mm^2}{1\ cm^2}$ or $\dfrac{1\ cm^2}{100\ mm^2}$
square centimeters and square decimeters	$1\ dm^2 = 100\ cm^2$	$\dfrac{100\ cm^2}{1\ dm^2}$ or $\dfrac{1\ dm^2}{100\ cm^2}$
square decimeters and square meters	$1\ m^2 = 100\ dm^2$	$\dfrac{100\ dm^2}{1\ m^2}$ or $\dfrac{1\ m^2}{100\ dm^2}$
square meters and ares (a)	$1\ a = 100\ m^2$	$\dfrac{100\ m^2}{1\ a}$ or $\dfrac{1\ a}{100\ m^2}$
ares and hectares (ha)	$1\ ha = 100\ a$	$\dfrac{100\ a}{1\ ha}$ or $\dfrac{1\ ha}{100\ a}$

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Example 6

How many square millimeters are in 1 square meter?

Solution:
We start with 1 m² and end up with square millimeters.

$$1 \text{ m}^2 = 1 \ \cancel{\text{m}^2} \times \frac{100 \ \cancel{\text{dm}^2}}{1 \ \cancel{\text{m}^2}} \times \frac{100 \ \cancel{\text{cm}^2}}{1 \ \cancel{\text{dm}^2}} \times \frac{100 \text{ mm}^2}{1 \ \cancel{\text{cm}^2}}$$

$$= 100 \times 100 \times 100 \text{ mm}^2$$

$$= 1{,}000{,}000 \text{ mm}^2$$

c Volume: The U.S. System

Volume: The U.S. System

Table 3 lists the units of volume in the U.S. system and their conversion factors.

TABLE 3		
UNITS OF VOLUME IN THE U.S. SYSTEM		
The relationship between	**is**	**To convert one to the other, multiply by**
cubic inches (in³) and cubic feet (ft³)	$1 \text{ ft}^3 = 1{,}728 \text{ in}^3$	$\dfrac{1{,}728 \text{ in}^3}{1 \text{ ft}^3}$ or $\dfrac{1 \text{ ft}^3}{1{,}728 \text{ in}^3}$
cubic feet and cubic yards (yd³)	$1 \text{ yd}^3 = 27 \text{ ft}^3$	$\dfrac{27 \text{ ft}^3}{1 \text{ yd}^3}$ or $\dfrac{1 \text{ yd}^3}{27 \text{ ft}^3}$
fluid ounces (fl oz) and pints (pt)	$1 \text{ pt} = 16 \text{ fl oz}$	$\dfrac{16 \text{ fl oz}}{1 \text{ pt}}$ or $\dfrac{1 \text{ pt}}{16 \text{ fl oz}}$
pints and quarts (qt)	$1 \text{ qt} = 2 \text{ pt}$	$\dfrac{2 \text{ pt}}{1 \text{ qt}}$ or $\dfrac{1 \text{ qt}}{2 \text{ pt}}$
quarts and gallons (gal)	$1 \text{ gal} = 4 \text{ qt}$	$\dfrac{4 \text{ qt}}{1 \text{ gal}}$ or $\dfrac{1 \text{ gal}}{4 \text{ qt}}$

Example 7

What is the capacity (volume) in pints of a 1-gallon container of milk?

Solution:

We change from gallons to quarts and then quarts to pints by multiplying by the appropriate conversion factors as given in Table 3.

$$1 \text{ gal} = 1 \text{ gal} \times \frac{4 \text{ qt}}{1 \text{ gal}} \times \frac{2 \text{ pt}}{1 \text{ qt}} = 1 \times 4 \times 2 \text{ pt} = 8 \text{ pt}$$

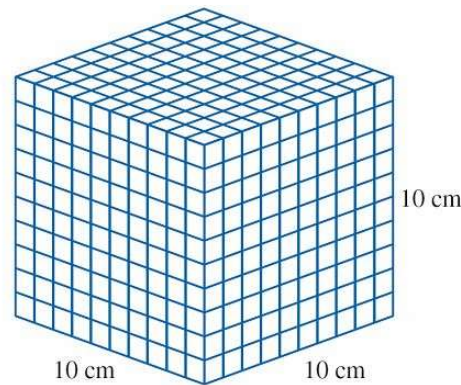A 1-gallon container has the same capacity as 8 one-pint containers.

D Volume: The Metric System

Volume: The Metric System

In the metric system the basic unit of measure for volume is the liter.

A liter is the volume enclosed by a cube that is 10 cm on each edge, as shown in Figure 2. We can see that a liter is equivalent to 1,000 cm^3.

1 liter = 10 cm × 10 cm × 10 cm
= 1,000 cm^3

Figure 2

Volume: The Metric System

The other units of volume in the metric system use the same prefixes we encountered previously.

The units with prefixes centi, deci, and deka are not as common as the others, so in Table 4 we include only liters, milliliters, hectoliters, and kiloliters.

TABLE 4 METRIC UNITS OF VOLUME		
The relationship between	**is**	**To convert one to the other, multiply by**
milliliters (mL) and liters	1 liter (L) = 1,000 mL	$\dfrac{1{,}000\text{ mL}}{1\text{ liter}}$ or $\dfrac{1\text{ liter}}{1{,}000\text{ mL}}$
hectoliters (hL) and liters	100 liters = 1 hL	$\dfrac{100\text{ liters}}{1\text{ hL}}$ or $\dfrac{1\text{ hL}}{100\text{ liters}}$
kiloliters (kL) and liters	1,000 liters (L) = 1 kL	$\dfrac{1{,}000\text{ liters}}{1\text{ kL}}$ or $\dfrac{1\text{ kL}}{1{,}000\text{ liters}}$

Example 9

A sports car has a 2.2-liter engine. What is the displacement (volume) of the engine in milliliters?

Solution:
Using the appropriate conversion factor from Table 4, we have

$$2.2 \text{ liters} = 2.2 \text{ liters} \times \frac{1{,}000 \text{ mL}}{1 \text{ liter}}$$

$$= 2.2 \times 1{,}000 \text{ mL}$$

$$= 2{,}200 \text{ mL}$$