Measurement

8

SECTION 8.1

Unit Analysis I: Length

Objectives

A Convert between lengths in the U.S. system.

B Convert between lengths in the metric system.

C Solve application problems involving unit analysis.

A U.S. Units of Length

U.S. Units of Length

Measuring the *length* of an object is done by assigning a number to its length.

To let other people know what that number represents, we include with it a unit of measure.

The most common units used to represent length in the U.S. system are inches, feet, yards, and miles.

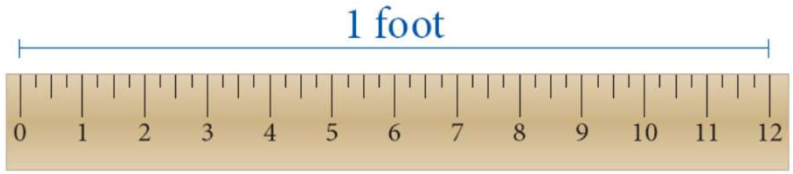
The basic unit of length is the foot.

U.S. Units of Length

The other units are defined in terms of feet, as Table 1 shows.

TABLE 1

12 inches (in.) =	1 foot (ft)
1 yard (yd) =	3 feet
1 mile (mi) =	5,280 feet



As you can see from the table, the abbreviations for inches, feet, yards, and miles are in., ft, yd, and mi, respectively.

What we haven't indicated, even though you may not have realized it, is what 1 foot represents.

U.S. Units of Length

We have defined all our units associated with length in terms of feet, but we haven't said what a foot is.

There is a long history of the evolution of what is now called a foot.

At different times in the past, a foot has represented different arbitrary lengths.

Currently, a foot is defined to be exactly 0.3048 meter (the basic measure of length in the metric system), where a meter is 1,650,763.73 wavelengths of the orange-red line in the spectrum of krypton-86 in a vacuum.

U.S. Units of Length

The reason a foot and a meter are defined this way is that we always want them to measure the same length.

Because the wavelength of the orange-red line in the spectrum of krypton-86 will always remain the same, so will the length that a foot represents.

Example 1

Convert 5 feet to inches.

Solution:

Because 1 foot = 12 inches, we can multiply 5 by 12 inches to get

$$5 \text{ feet} = 5 \times 12 \text{ inches}$$

$$= 60 \text{ inches}$$

U.S. Units of Length

This method of converting from feet to inches probably seems fairly simple.

For more complicated problems, we need another way to show conversions so that we can be certain to end them with the correct unit of measure.

For example, since 1 ft = 12 in., we can say that there are 12 in. per 1 ft or 1 ft per 12 in.

That is,

$$\frac{12 \text{ in.}}{1 \text{ ft}} \longleftarrow \text{Per} \qquad \text{or} \qquad \frac{1 \text{ ft}}{12 \text{ in.}} \longleftarrow \text{Per}$$

U.S. Units of Length

We call the expressions $\frac{12 \text{ in.}}{1 \text{ ft}}$ and $\frac{1 \text{ ft}}{12 \text{ in.}}$ *conversion factors*.

The fraction bar is read as "per." Both these conversion factors are really just the number 1.

That is,

$$\frac{12 \text{ in.}}{1 \text{ ft}} = \frac{12 \text{ in.}}{12 \text{ in.}} = 1$$

We already know that multiplying a number by 1 leaves the number unchanged.

U.S. Units of Length

So, to convert from one unit to the other, we can multiply by one of the conversion factors without changing value.

Both the conversion factors above say the same thing about the units feet and inches.

They both indicate that there are 12 inches in every foot.

The one we choose to multiply by depends on what units we are starting with and what units we want to end up with.

U.S. Units of Length

If we start with feet and we want to end up with inches, we multiply by the conversion factor

$$\frac{12 \text{ in.}}{1 \text{ ft}}$$

The units of feet will divide out and leave us with inches.

$$\begin{aligned} 5 \text{ feet} &= 5 \cancel{\text{ft}} \times \frac{12 \text{ in.}}{1 \cancel{\text{ft}}} \\ &= 5 \times 12 \text{ in.} \\ &= 60 \text{ in.} \end{aligned}$$

The key to this method of conversion lies in setting the problem up so that the correct units divide out to simplify the expression.

U.S. Units of Length

We are treating units such as feet in the same way we treated factors when reducing fractions.

If a factor is common to the numerator and the denominator, we can divide it out and simplify the fraction.

The same idea holds for units such as feet.

U.S. Units of Length

We can rewrite Table 1 so that it shows the conversion factors associated with units of length, as shown in Table 2.

TABLE 1

12 inches (in.) =	1 foot (ft)
1 yard (yd) =	3 feet
1 mile (mi) =	5,280 feet

TABLE 2

UNITS OF LENGTH IN THE U.S. SYSTEM

The relationship between	is	To convert one to the other, multiply by
feet and inches	12 in. = 1 ft	$\frac{12 \text{ in.}}{1 \text{ ft}}$ or $\frac{1 \text{ ft}}{12 \text{ in.}}$
feet and yards	1 yd = 3 ft	$\frac{3 \text{ ft}}{1 \text{ yd}}$ or $\frac{1 \text{ yd}}{3 \text{ ft}}$
feet and miles	1 mi = 5,280 ft	$\frac{5{,}280 \text{ ft}}{1 \text{ mi}}$ or $\frac{1 \text{ mi}}{5{,}280 \text{ ft}}$

B Metric Units of Length

Metric Units of Length

In the metric system the standard unit of length is a meter. A meter is a little longer than a yard (about 3.4 inches longer). The other units of length in the metric system are written in terms of a meter.

The metric system uses prefixes to indicate what part of the basic unit of measure is being used.

For example, in *milli*meter the prefix *milli* means "one-thousandth" of a meter.

Metric Units of Length

Table 3 gives the meanings of the most common metric prefixes.

TABLE 3
THE MEANING OF METRIC PREFIXES

Prefix	Meaning
milli	0.001
centi	0.01
deci	0.1
deka	10
hecto	100
kilo	1,000

Metric Units of Length

We can use these prefixes to write the other units of length and conversion factors for the metric system, as given in Table 4.

TABLE 4

METRIC UNITS OF LENGTH

The relationship between	is	To convert one to the other, multiply by
millimeters (mm) and meters (m)	1,000 mm = 1 m	$\frac{1{,}000\text{ mm}}{1\text{ m}}$ or $\frac{1\text{ m}}{1{,}000\text{ mm}}$
centimeters (cm) and meters	100 cm = 1 m	$\frac{100\text{ cm}}{1\text{ m}}$ or $\frac{1\text{ m}}{100\text{ cm}}$
decimeters (dm) and meters	10 dm = 1 m	$\frac{10\text{ dm}}{1\text{ m}}$ or $\frac{1\text{ m}}{10\text{ dm}}$
dekameters (dam) and meters	1 dam = 10 m	$\frac{10\text{ m}}{1\text{ dam}}$ or $\frac{1\text{ dam}}{10\text{ m}}$
hectometers (hm) and meters	1 hm = 100 m	$\frac{100\text{ m}}{1\text{ hm}}$ or $\frac{1\text{ hm}}{100\text{ m}}$
kilometers (km) and meters	1 km = 1,000 m	$\frac{1{,}000\text{ m}}{1\text{ km}}$ or $\frac{1\text{ km}}{1{,}000\text{ m}}$

Metric Units of Length

We use the same method to convert between units in the metric system as we did with the U.S. system.

We choose the conversion factor that will allow the units we start with to divide out, leaving the units we want to end up with.

Example 4

Convert 25 millimeters to meters.

Solution:

To convert from millimeters to meters, we multiply by the conversion factor $\frac{1\text{ m}}{1{,}000\text{ mm}}$.

$$25\text{ mm} = 25\text{ mm} \times \frac{1\text{ m}}{1{,}000\text{ mm}}$$

$$= \frac{25\text{ m}}{1{,}000}$$

$$= 0.025\text{ m}$$

Metric Units of Length

The most common units of length in the metric system are millimeters, centimeters, meters, and kilometers. The other units of length we have listed in our table of metric lengths are not as widely used.

The method we have used to convert from one unit of length to another in Example 4 is called *unit analysis*. If you take a chemistry class, you will see it used many times. The same is true of many other science classes as well.

Metric Units of Length

We can summarize the procedure used in unit analysis with the following steps:

Strategy **Unit Analysis**

Step 1 Identify the units you are starting with.

Step 2 Identify the units you want to end with.

Step 3 Find conversion factors that will bridge the starting units and the ending units.

Step 4 Set up the multiplication problem so that all units except the units you want to end with will divide out.

C Applications

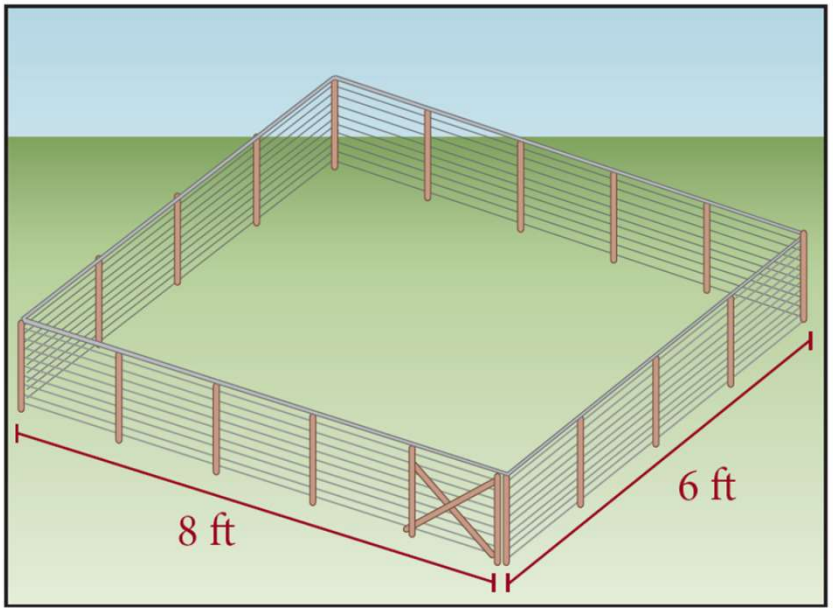
Example 6

A sheep rancher is making new lambing pens for the upcoming lambing season. Each pen is a rectangle 6 feet wide and 8 feet long. The fencing material he wants to use sells for $1.36 per foot.

If he is planning to build five separate lambing pens (they are separate because he wants a walkway between them), how much will he have to spend for fencing material?

Example 6 – *Solution*

To find the amount of fencing material he needs for one pen, we find the perimeter of a pen.



Perimeter = 6 + 6 + 8 + 8 = 28 feet

Example 6 – *Solution*

cont'd

We set up the solution to the problem using unit analysis.

Our starting unit is *pens* and our ending unit is *dollars*.

Here are the conversion factors that will form a bridge between pens and dollars.

1 pen = 28 feet of fencing

1 foot of fencing = 1.36 dollars

Example 6 – *Solution*

cont'd

Next we write the multiplication problem, using the conversion factors, that will allow all the units except dollars to divide out.

$$5 \text{ pens} = 5 \text{ pens} \times \frac{28 \text{ feet of fencing}}{1 \text{ pen}} \times \frac{1.36 \text{ dollars}}{1 \text{ foot of fencing}}$$

$$= 5 \times 28 \times 1.36 \text{ dollars}$$

$$= \$190.40$$