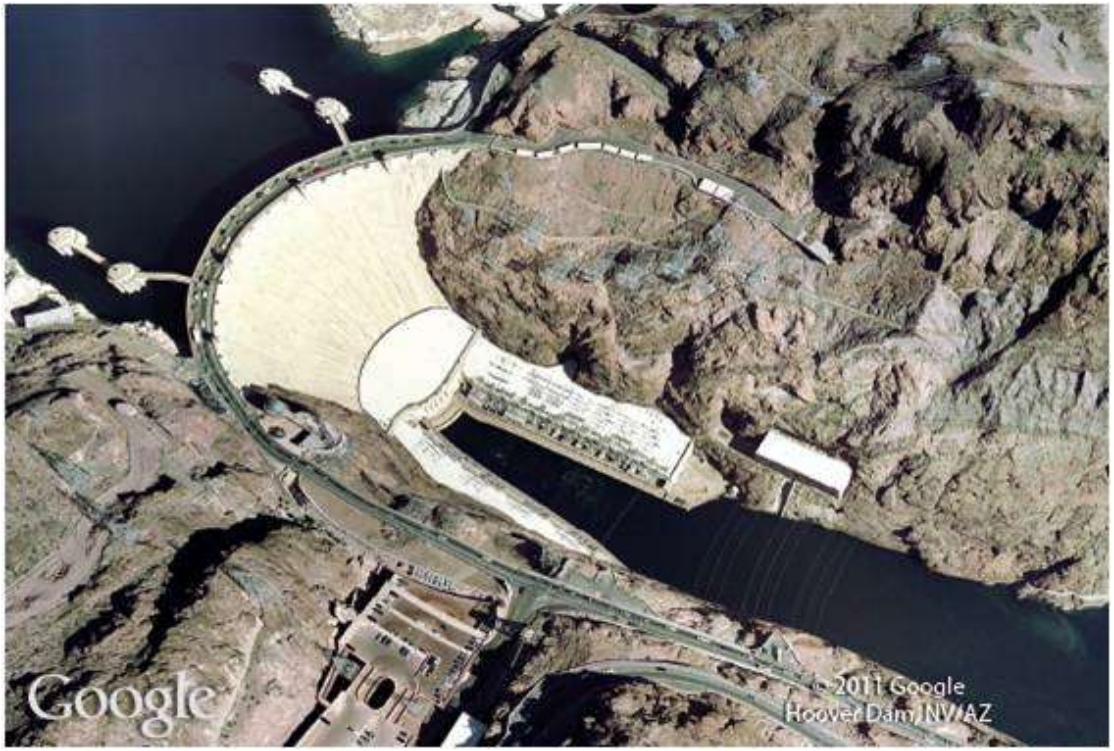
Percent

7



SECTION 7.2

Basic Percent Problems

Objectives

A Solve percent problems using equations.

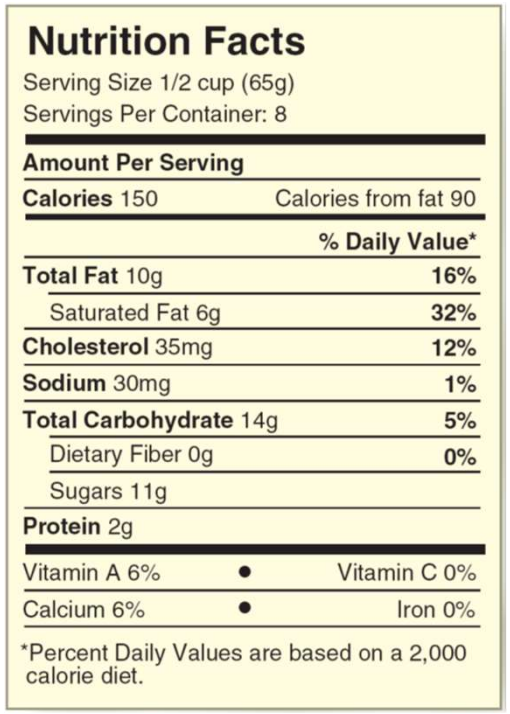
B Solve percent problems using proportions.

Basic Percent Problems

The American Dietetic Association (ADA) recommends eating foods in which the number of calories from fat is less than 30% of the total number of calories.

Foods that satisfy this requirement are considered healthy foods.

Is the nutrition label shown at the right from a food that the ADA would consider healthy?



Nutrition Facts

Serving Size 1/2 cup (65g)
Servings Per Container: 8

Amount Per Serving	
Calories 150	Calories from fat 90
	% Daily Value*
Total Fat 10g	**16%**
Saturated Fat 6g	**32%**
Cholesterol 35mg	**12%**
Sodium 30mg	**1%**
Total Carbohydrate 14g	**5%**
Dietary Fiber 0g	**0%**
Sugars 11g	
Protein 2g	
Vitamin A 6% •	Vitamin C 0%
Calcium 6% •	Iron 0%

*Percent Daily Values are based on a 2,000 calorie diet.

Nutrition label from vanilla ice cream

Figure 1

This is the type of question we will be able to answer after we have worked through the examples in this section.

A Solving Percent Problems Using Equations

Solving Percent Problems Using Equations

This section is concerned with three kinds of word problems that are associated with percents.

Here is an example of each type:

Type A: What number is 15% of 63?
Type B: What percent of 42 is 21?
Type C: 25 is 40% of what number?

Solving Percent Problems Using Equations

The first method we use to solve all three types of problems involves translating the sentences into equations and then solving the equations.

The following translations are used to write the sentences as equations:

English	Mathematics
is	$=$
of	$\cdot$ (multiply)
a number	n
what number	n
what percent	n

Solving Percent Problems Using Equations

The word *is* always translates to an = sign.

The word *of* almost always means multiply.

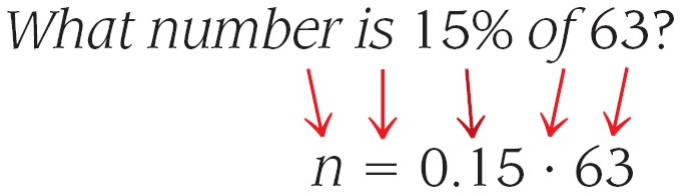
The number we are looking for can be represented with a letter, such as *n* or *x*.

Example 1

What number is 15% of 63?

Solution:

We translate the sentence into an equation as follows:



To do arithmetic with percents, we have to change to decimals.

That is why 15% is rewritten as 0.15.

Example 1 – *Solution*

cont'd

Solving the equation, we have

$$n = 0.15 \cdot 63$$

$$n = 9.45$$

Therefore, 15% of 63 is 9.45.

Example 2

What percent of 42 is 21?

Solution:

We translate the sentence as follows:

$$\begin{array}{c} \textit{What percent of } 42 \textit{ is } 21? \\ \downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow \\ n \cdot 42 = 21 \end{array}$$

We solve for n by dividing both sides by 42.

$$\frac{n \cdot \cancel{42}}{\cancel{42}} = \frac{21}{42}$$

Example 2 – *Solution*

cont'd

$$n = \frac{21}{42}$$

$$n = 0.50$$

Because the original problem asked for a percent, we change 0.50 to a percent:

$$n = 50\%$$

Therefore, 21 is 50% of 42.

Example 3

25 is 40% of what number?

Solution:

Following the procedure from the first two examples, we have

$$25 \text{ is } 40\% \text{ of what number?}$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$25 = 0.40 \cdot n$$

Again, we changed 40% to 0.40 so we can do the arithmetic involved in the problem.

Example 3 – *Solution*

cont'd

Dividing both sides of the equation by 0.40, we have

$$\frac{25}{0.40} = \frac{\cancel{0.40} \cdot n}{\cancel{0.40}}$$

$$\frac{25}{0.40} = n$$

$$62.5 = n$$

Therefore, 25 is 40% of 62.5.

Solving Percent Problems Using Equations

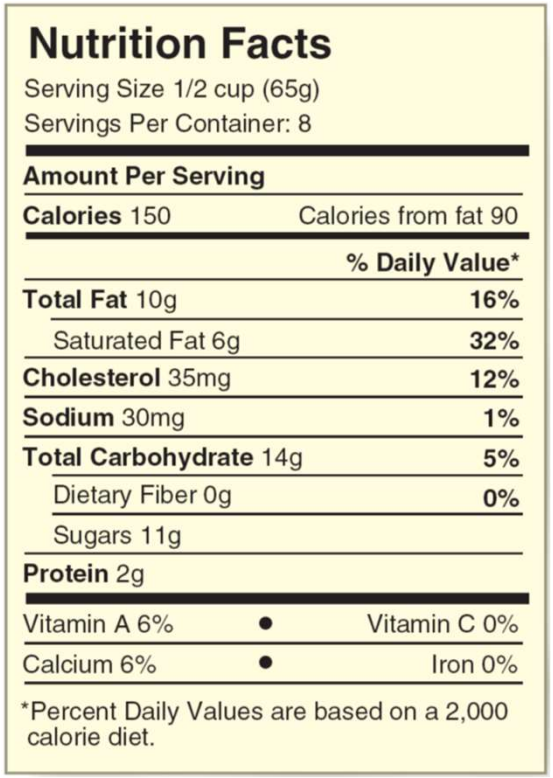
As you can see, all three types of percent problems are solved in a similar manner. We write *is* as =, *of* as •, and *what number* as *n*.

The resulting equation is then solved to obtain the answer to the original question.

Example 7

As we mentioned in the introduction to this section, the American Dietetic Association recommends eating foods in which the number of calories from fat is less than 30% of the total number of calories.

According to the nutrition label, what percent of the total number of calories is fat calories?



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Nutrition label from vanilla ice cream

Example 7 – *Solution*

To solve this problem, we must write the question in the form of one of the three basic percent problems.

Because there are 90 calories from fat and a total of 150 calories, we can write the question this way: 90 is what percent of 150?

Now that we have written the question in the form of one of the basic percent problems, we simply translate it into an equation.

Example 7 – *Solution*

cont'd

Then we solve the equation.

$$90 \text{ is what percent of } 150?$$

$$90 = n \cdot 150$$

$$\frac{90}{150} = n$$

$$n = 0.60 = 60\%$$

The number of calories from fat in this package of ice cream is 60% of the total number of calories.

Thus the ADA would not consider this to be a healthy food.

B Solving Percent Problems Using Proportions

Solving Percent Problems Using Proportions

We can look at percent problems in terms of proportions also.

For example, we know that 24% is the same as $\frac{24}{100}$, which reduces to $\frac{6}{25}$.

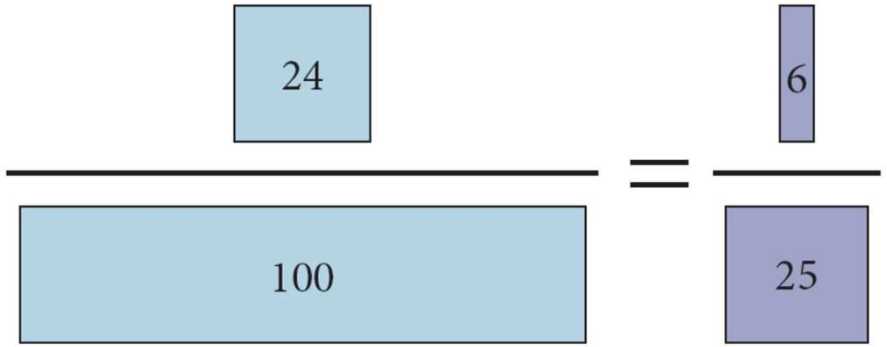
That is

$$\frac{24}{100} = \frac{6}{25}$$

24 is to 100 as 6 is to 25

Solving Percent Problems Using Proportions

We can illustrate this visually with boxes of proportional lengths:



In general, we say

$$\frac{\text{Percent}}{100} = \frac{\text{Amount}}{\text{Base}}$$

Percent is to 100 as amount is to base.

Example 8

What number is 15% of 63?

Solution:
We let *n* be the number in question.

We reason that *n* will be smaller than 63 because it is only 15% of 63.

The base is 63 and the amount is *n*.

We compare *n* to 63 as we compare 15 to 100.

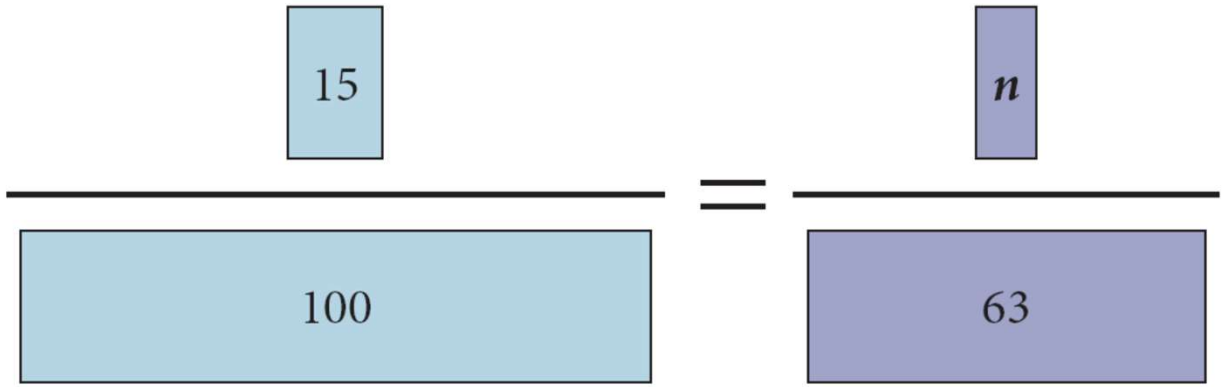
Example 8 – *Solution*

cont'd

Our proportion sets up as follows:

$$\underbrace{15 \text{ is to } 100}_{\downarrow} \quad \underset{\downarrow}{\text{as}} \quad \underbrace{n \text{ is to } 63}_{\downarrow}$$

$$\frac{15}{100} = \frac{n}{63}$$



Example 8 – *Solution*

cont'd

Solving the proportion, we have

$$15 \cdot 63 = 100n$$ Extremes/means property

$$945 = 100n$$ Simplify the left side.

$$9.45 = n$$ Divide each side by 100.