# Ratio and Proportion

# 6

SECTION 6.5

# Similar Figures

# Objectives

- **A** Use proportions to find the lengths of sides of similar triangles.
- **B** Use proportions to find the lengths of sides of other similar figures.
- **C** Draw a figure similar to a given figure, given the length of one side.
- **D** Use similar figures to solve application problems.

# Similar Figures

This 8-foot-high bronze sculpture “Cellarman” in Napa, California, is an exact replica of the smaller, 12-inch sculpture.

Both pieces are the product of artist Tim Lloyd of Arroyo Grande, California.

# Similar Figures

In mathematics, when two or more objects have the same shape, but are different sizes, we say they are similar.

If two figures are similar, then their corresponding sides are proportional.

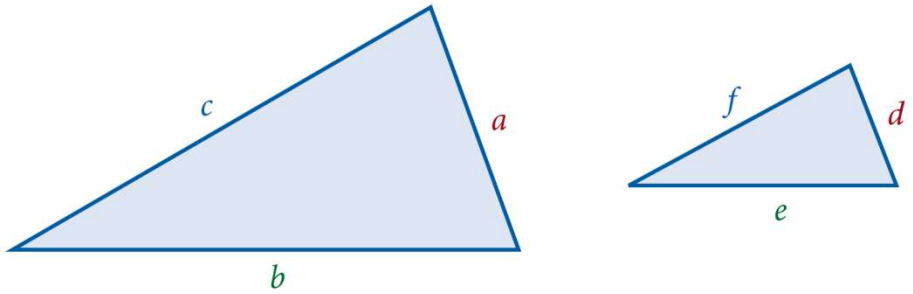
In order to give more details on what we mean by corresponding sides of similar figures, it will be helpful to introduce a simple way to label the parts of a triangle.

# A Similar Triangles

# Similar Triangles

Two triangles that have the same shape are similar when their corresponding sides are proportional, or have the same ratio. The triangles shown are similar.

| Corresponding Sides | Ratio |
| --- | --- |
| Side $a$ corresponds with side $d$. | $\frac{a}{d}$ |
| Side $b$ corresponds with side $e$. | $\frac{b}{e}$ |
| Side $c$ corresponds with side $f$. | $\frac{c}{f}$ |

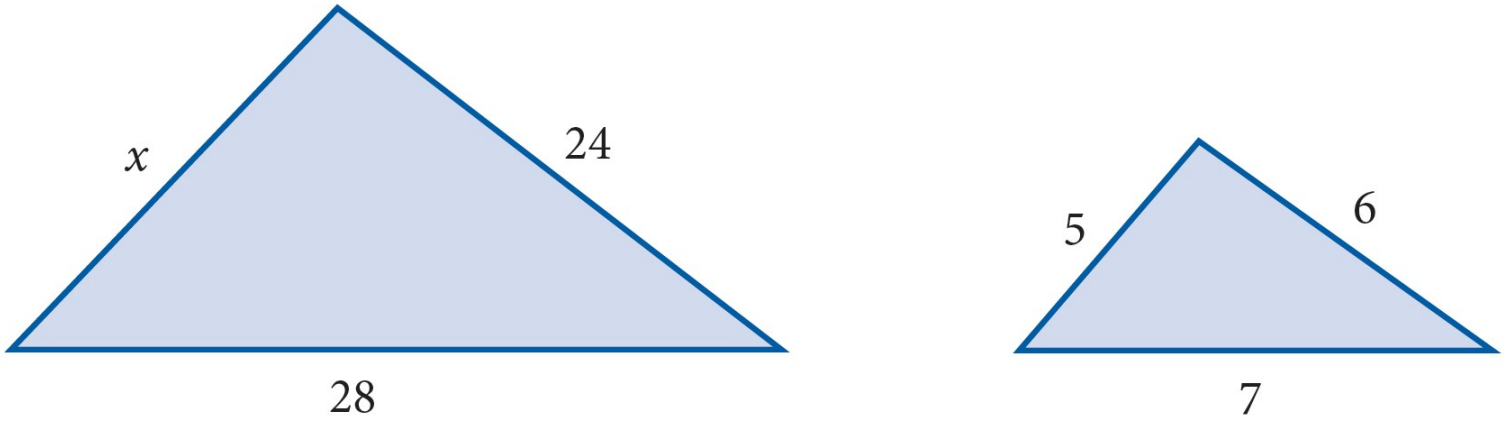



Because their corresponding sides are proportional, we write

$$\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$$

# Example 1

The two triangles below are similar. Find side *x*.




Solution:

To find the length *x*, we set up a proportion of equal ratios.

The ratio of *x* to 5 is equal to the ratio of 24 to 6 and to the ratio of 28 to 7.

# Example 1 – *Solution*

cont'd

Algebraically we have

$$\frac{x}{5} = \frac{24}{6} \quad \text{and} \quad \frac{x}{5} = \frac{28}{7}$$

We can solve either proportion to get our answer.

The first gives us

$$\frac{x}{5} = 4 \qquad \frac{24}{6} = 4$$

$$x = 4 \cdot 5 \qquad \text{Multiply both sides by 5.}$$

$$x = 20 \qquad \text{Simplify.}$$

# B Other Similar Figures

# Other Similar Figures

When one shape or figure is either a reduced or enlarged copy of the same shape or figure, we consider them similar.

For example, video viewed over the Internet was once confined to a small “postage stamp” size.

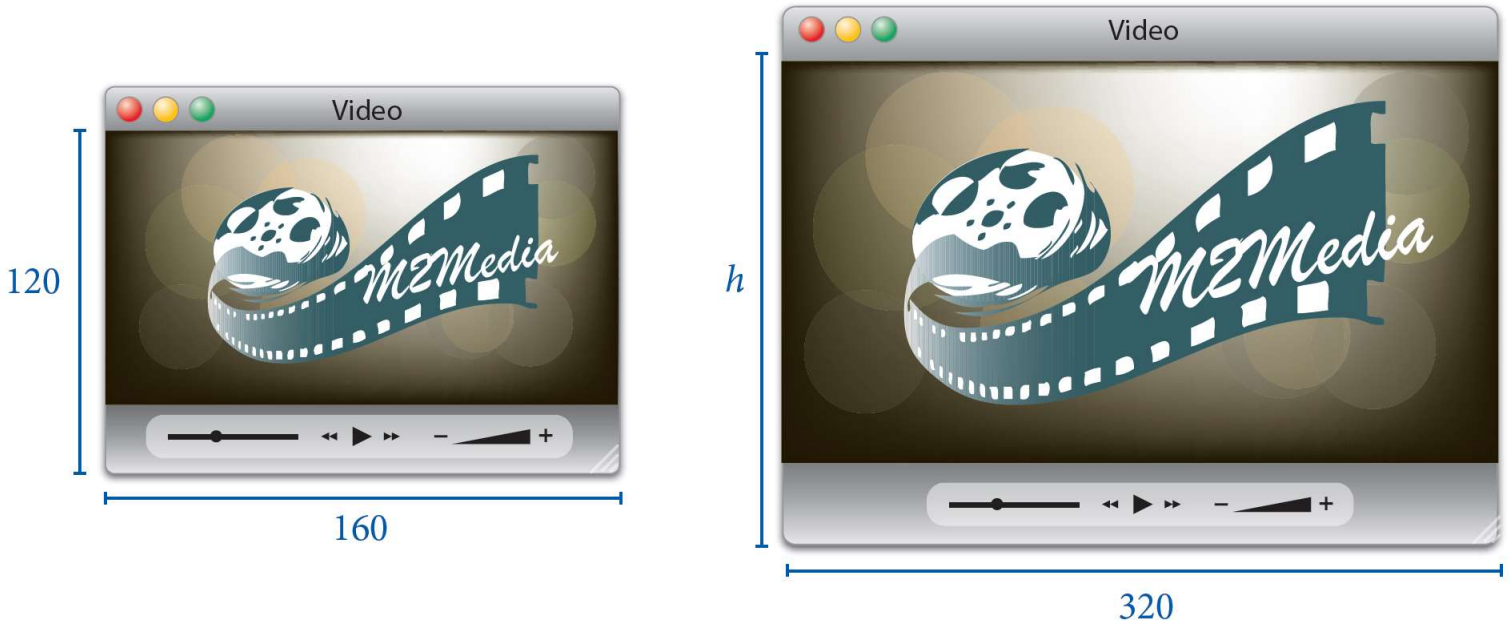
Now it is common to see larger video over the Internet.

Although the width and height have increased, the shape of the video has not changed.

# Example 2

The width and height of the two video clips are proportional.

Find the height, *h*, in pixels of the larger video window.



# Example 2 – *Solution*

We write our proportion as the ratio of the height of the new video to the height of the old video is equal to the ratio of the width of the new video to the width of the old video.

$$\frac{h}{120} = \frac{320}{160}$$

$$\frac{h}{120} = 2$$

$$h = 2 \cdot 120$$

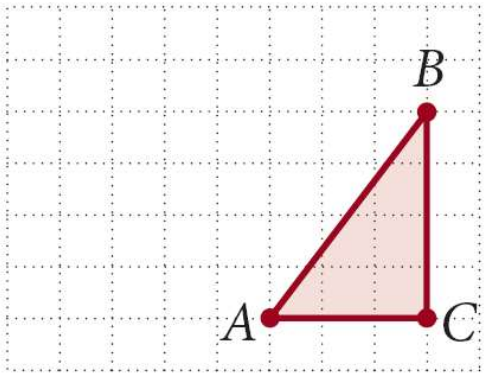
$$h = 240$$

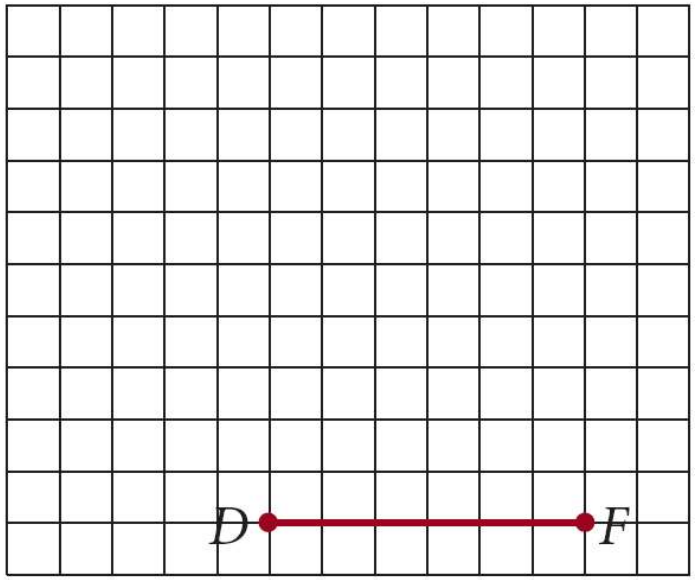
The height of the larger video is 240 pixels.

# C Drawing Similar Figures

# Example 3

Draw a triangle similar to triangle *ABC*, if *AC* is proportional to *DF*. Make *E* the third vertex of the new triangle.



# Example 3 – *Solution*

We see that *AC* is 3 units in length and *BC* has a length of 4 units.

Since *AC* is proportional to *DF*, which has a length of 6 units, we set up a proportion to find the length *EF*.

$$\frac{EF}{BC} = \frac{DF}{AC}$$

$$\frac{EF}{4} = \frac{6}{3}$$

$$\frac{EF}{4} = 2$$

$$EF = 8$$

# Example 3 – *Solution*

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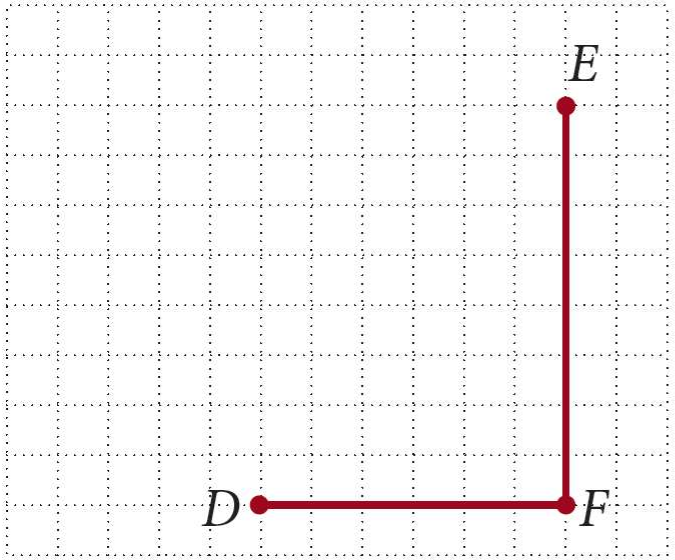
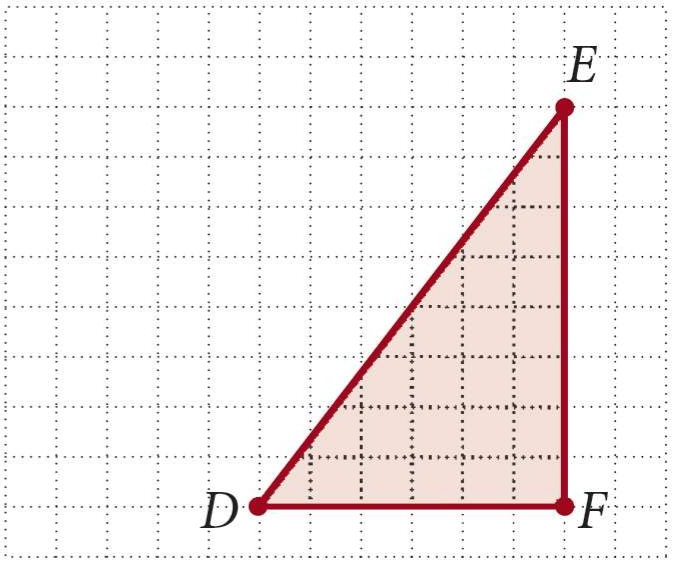
Now we can draw *EF* with a length of 8 units, then complete the triangle by drawing line *DE*.



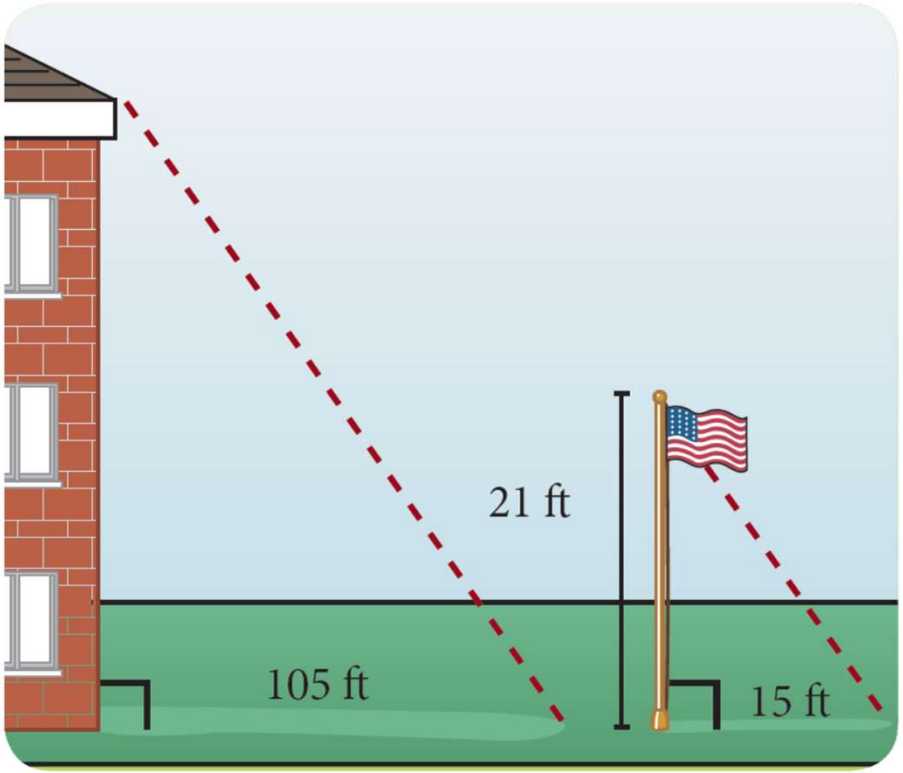




We have drawn triangle *DEF* similar to triangle *ABC*.

# D Applications

# Example 4

A building casts a shadow of 105 feet while a 21-foot flagpole casts a shadow that is 15 feet. Find the height of the building.



# Example 4 – *Solution*

The figure shows both the building and the flagpole, along with their respective shadows.

From the figure it is apparent that we have two similar triangles.

Letting $x$ = the height of the building, we have

$$\frac{x}{21} = \frac{105}{15}$$

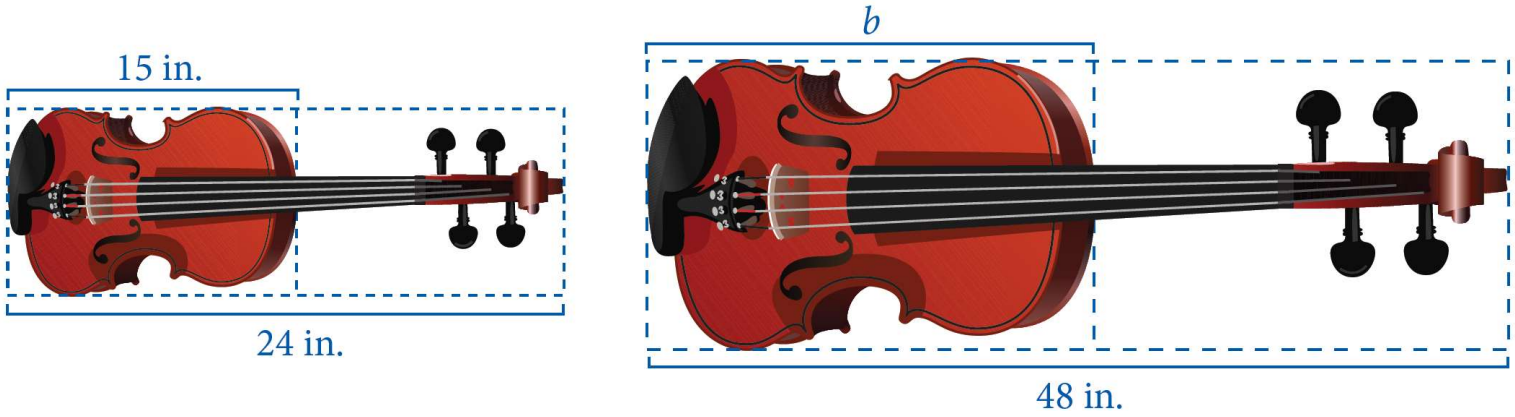
$$15x = 2205 \qquad \text{Extremes/means property}$$

$$x = 147 \qquad \text{Divide both sides by 15.}$$

The height of the building is 147 feet.

# Example 5

The instruments in the violin family include the bass, cello, viola, and violin. These instruments can be considered similar figures because the entire length of each instrument is proportional to its body length.




The entire length of a violin is 24 inches, while the body length is 15 inches. Find the body length of a cello if the entire length is 48 inches.

# Example 5 – *Solution*

Let $b$ equal the body length of the cello, and set up the proportion.

$$\frac{b}{15} = \frac{48}{24}$$

$$\frac{b}{15} = 2$$

$$b = 2 \cdot 15$$

$$b = 2 \cdot 15$$

$$b = 30$$

The body length of a cello is 30 inches.