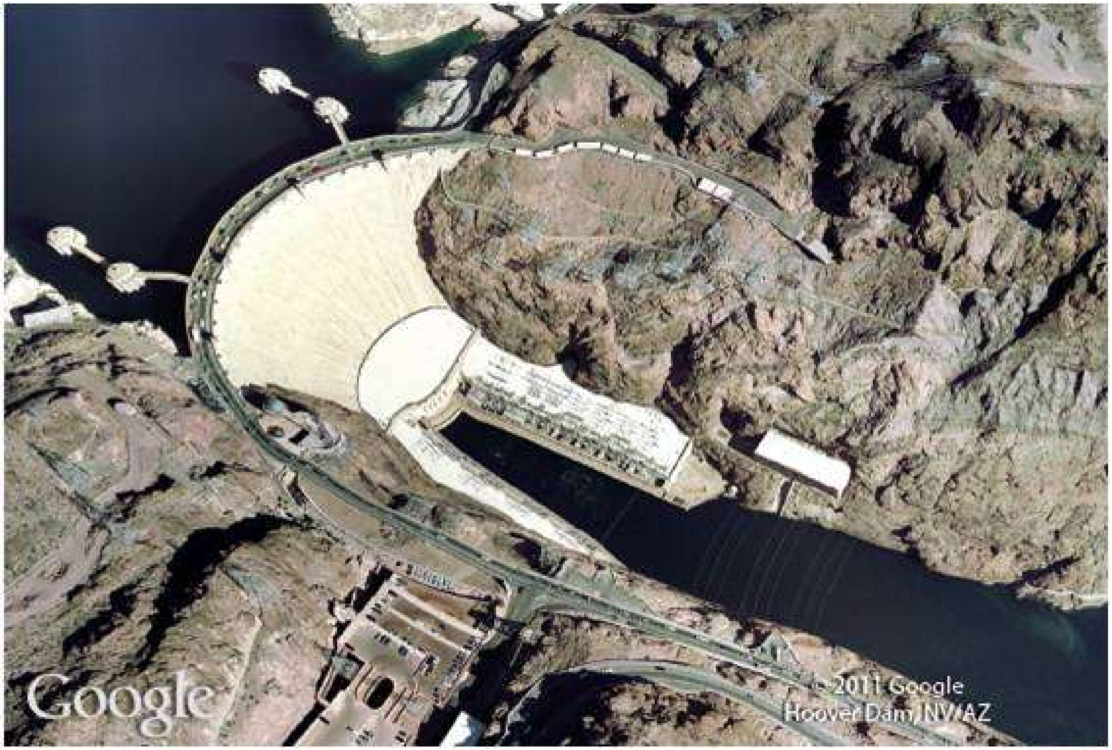
Decimals

5



SECTION 5.9

Adding and Subtracting Roots

Objective

A Add and subtract expressions containing square roots.

A Simplifying Radical Expressions

Simplifying Radical Expressions

Suppose a surveying team must choose between two mountains through which to dig a tunnel.

Remember, to calculate the length of a tunnel, the team must form a right angle by connecting lines from each end of the tunnel.

The first mountain has one connecting line that measures 3 miles, and the other is 6 miles.

The second mountain has one connecting line that measures 4 miles, and the other is 8 miles.

Simplifying Radical Expressions

Refer Figure 1 and Figure 2.

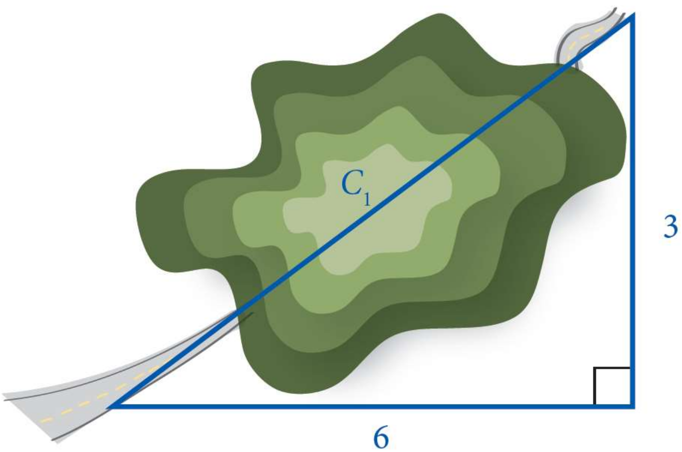


Figure 1

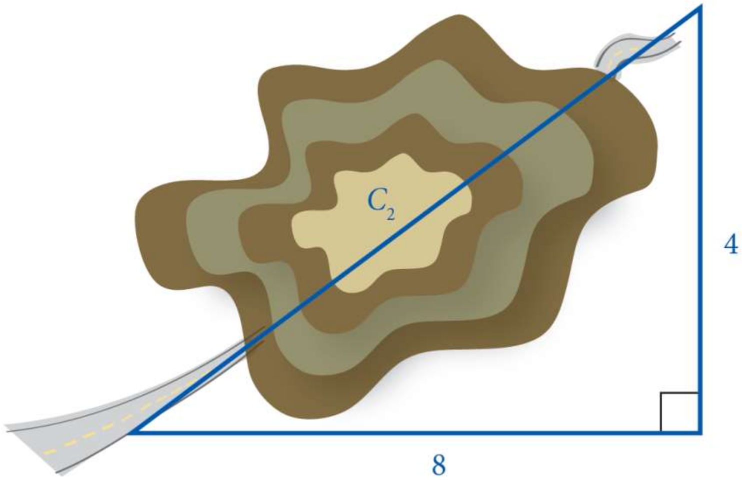


Figure 2

What is the difference in lengths of the two tunnels? To solve this problem, we must understand how to add and subtract roots.

Simplifying Radical Expressions

The surveying team will choose to dig one of the two tunnels, represented by the hypotenuse *c* of each triangle.

In order to determine the length of each hypotenuse, we need to apply the Pythagorean theorem, which involves square roots.

Hypotenuse for Figure 1	Hypotenuse for Figure 2
$c_1 = \sqrt{3^2 + 6^2}$	$c_2 = \sqrt{4^2 + 8^2}$
$c_1 = \sqrt{9 + 36}$	$c_2 = \sqrt{16 + 64}$
$c_1 = \sqrt{45}$	$c_2 = \sqrt{80}$
$c_1 = \sqrt{9 \cdot 5}$	$c_2 = \sqrt{16 \cdot 5}$
$c_1 = 3\sqrt{5}$	$c_2 = 4\sqrt{5}$

Simplifying Radical Expressions

The hypotenuse for Figure 1 is $3\sqrt{5}$ miles long, and the hypotenuse for Figure 2 is $4\sqrt{5}$ miles long.

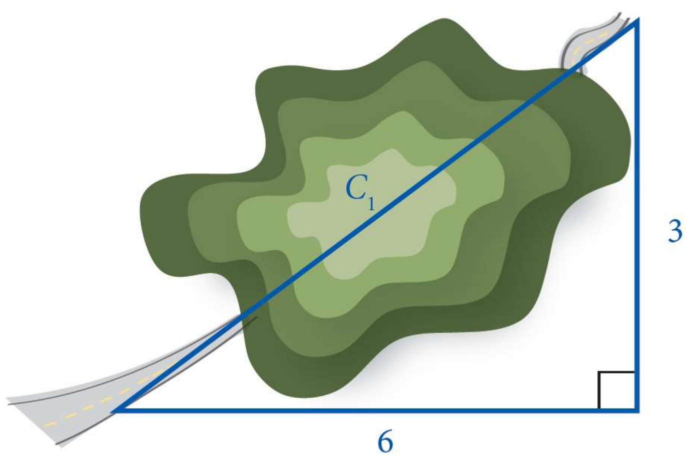


Figure 1

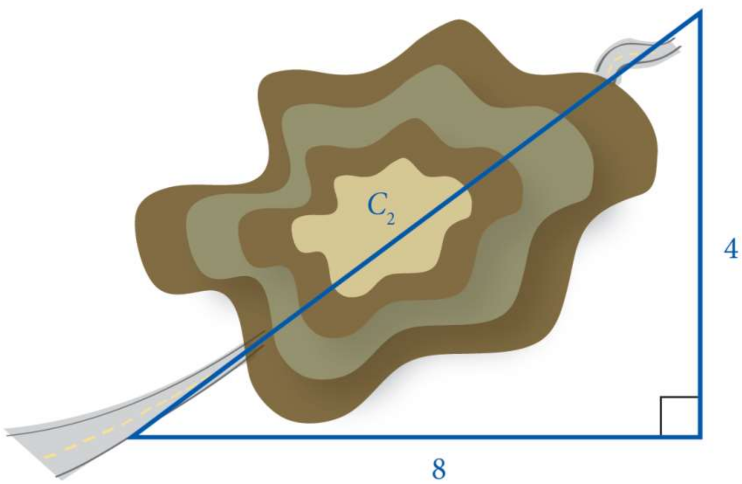


Figure 2

Simplifying Radical Expressions

Now we have our values to answer the question, “What is the difference in lengths of the two tunnels?” As you can see, both lengths contain a radical.

We add and subtract radical expressions in the same way we add and subtract similar terms, that is, by applying the distributive property.

Simplifying Radical Expressions

Here is how we use the distributive property to subtract $3\sqrt{5}$ from $4\sqrt{5}$:

$$4\sqrt{5} - 3\sqrt{5} = (4 - 3)\sqrt{5} \quad \text{Distributive property}$$

$$= \sqrt{5} \quad \text{Subtract 3 from 4.}$$

Here is the formal definition for a radical expression:

Definition

A **radical expression** is an expression that contains a radical, as well as any combination of numbers, variables, operation symbols, and grouping symbols. This definition includes the use of exponents and fractions.

Simplifying Radical Expressions

Here are some more examples that will show how we use the distributive property to combine radical expressions. (Assume all variables represent positive numbers.)

Examples

Combine using the distributive property. (Assume all variables represent positive numbers.)

1. $5\sqrt{6} - 2\sqrt{6} = (5 - 2)\sqrt{6} = 3\sqrt{6}$

2. $6\sqrt{x} + 3\sqrt{x} = (6 + 3)\sqrt{x} = 9\sqrt{x}$

3. $4\sqrt{3} + 3\sqrt{3} - 2\sqrt{3} = (4 + 3 - 2)\sqrt{3} = 5\sqrt{3}$

Simplifying Radical Expressions

Next, suppose we try to add $\sqrt{12}$ and $\sqrt{75}$.

A calculator can help us decide if $\sqrt{12} + \sqrt{75}$ is the same as $\sqrt{87}$. Here are the decimal approximations a calculator will give us:

$$\sqrt{12} + \sqrt{75} = 3.4641016 + 8.6602540 = 12.1243556$$
$$\sqrt{12 + 75} = \sqrt{87} = 9.3273791$$

As you can see, the two results are quite different, so we can assume that it would be a mistake to add the numbers under the square roots. That is,

$$\sqrt{12} + \sqrt{75} \neq \sqrt{12 + 75}$$

Simplifying Radical Expressions

The correct way to add $\sqrt{12}$ and $\sqrt{75}$ is to simplify each expression by taking as much out from under each square root as possible.

Then, if the square roots in the resulting expressions are the same, we can add using the distributive property. Here is the way the problem is done correctly:

$$\begin{aligned} \sqrt{12} + \sqrt{75} &= \sqrt{2 \cdot 2 \cdot 3} + \sqrt{5 \cdot 5 \cdot 3} && \text{Simplify each square root.} \\ &= 2\sqrt{3} + 5\sqrt{3} \\ &= (2 + 5)\sqrt{3} && \text{Distributive property} \\ &= 7\sqrt{3} && \text{Add 2 and 5.} \end{aligned}$$

Simplifying Radical Expressions

On a calculator, $7\sqrt{3} = 7(1.7320508) = 12.1243556$, which matches the approximation a calculator gives for $\sqrt{12} + \sqrt{75}$.

Example 4

Combine, if possible: $\sqrt{18} + \sqrt{50} - \sqrt{8}$

Solution:

First we simplify each term by taking as much out from under the square root as possible. Then we use the distributive property to combine terms if they contain the same square root.

$$\begin{aligned}\sqrt{18} + \sqrt{50} - \sqrt{8} &= \sqrt{3 \cdot 3 \cdot 2} + \sqrt{5 \cdot 5 \cdot 2} - \sqrt{2 \cdot 2 \cdot 2} \\ &= 3\sqrt{2} + 5\sqrt{2} - 2\sqrt{2} \\ &= (3 + 5 - 2)\sqrt{2} \\ &= 6\sqrt{2}\end{aligned}$$

Example 6

Assume *x* is a positive number and combine, if possible:

$$5\sqrt{12x^3} - 3\sqrt{75x^3}$$

Solution:

We simplify each square root, and then we subtract.

$$5\sqrt{12x^3} - 3\sqrt{75x^3} = 5\sqrt{2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x} - 3\sqrt{5 \cdot 5 \cdot 3 \cdot x \cdot x \cdot x}$$

$$= 5 \cdot 2 \cdot x\sqrt{3x} - 3 \cdot 5 \cdot x\sqrt{3x}$$

$$= 10x\sqrt{3x} - 15x\sqrt{3x}$$

$$= -5x\sqrt{3x}$$