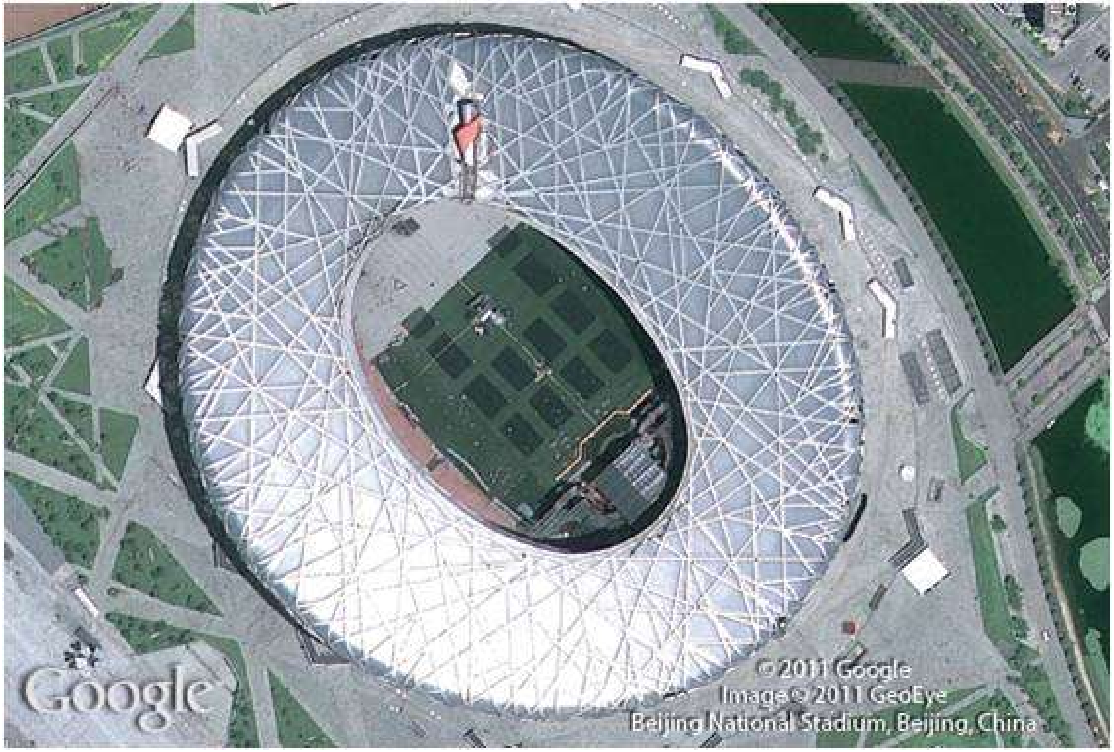
Decimals

5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SECTION 5.8

Simplifying Square Roots

Objective

A Use the multiplication property to simplify square roots.

A Simplifying Square Roots

Simplifying Square Roots

Do you know that $\sqrt{50}$ and $5 \cdot \sqrt{2}$ are the same number? One way to convince yourself that this is true is with a calculator.

Although a calculator will give the same result for both $\sqrt{50}$ and $5 \cdot \sqrt{2}$, it does not tell us *why* the answers are the same. The discussion below shows why the results are the same.

First, notice that the expressions $\sqrt{4 \cdot 9}$ and $\sqrt{4} \cdot \sqrt{9}$ have the same value:

$$\sqrt{4 \cdot 9} = \sqrt{36} = 6 \quad \text{and} \quad \sqrt{4} \cdot \sqrt{9} = 2 \cdot 3 = 6$$

Simplifying Square Roots

Both are equal to 6. When we are multiplying and taking square roots, we can either multiply first and then take the square root of what we get, or we can take square roots first and then multiply. This is called the *multiplication property for square roots*. In symbols, we write the property this way:

Multiplication Property for Square Roots

If a and b are positive numbers, then

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

In words: The square root of a product is the product of the square roots.

Simplifying Square Roots

Also, when a number occurs twice as a factor in a square root, then the square root simplifies to just that number.

For example, $\sqrt{5 \cdot 5}$ is really $\sqrt{25}$, which is the same as just 5.

Therefore, $\sqrt{5 \cdot 5} = 5$.

Repeated Factor Property for Square Roots

If a is a positive number, then

$$\sqrt{a \cdot a} = a$$

Simplifying Square Roots

But how do these two properties help us simplify expressions such as $\sqrt{50}$. To see the answer to this question, we must factor 50 into the product of its prime factors:

$$\sqrt{50} = \sqrt{5 \cdot 5 \cdot 2}$$

The factor 5 occurs twice, meaning that we have a perfect square (5 • 5 = 25) under the radical. Writing this as two separate square roots, we have

$$\sqrt{5 \cdot 5 \cdot 2} = \sqrt{5 \cdot 5} \cdot \sqrt{2}$$

$$= 5 \cdot \sqrt{2}$$

Simplifying Square Roots

Rule **Square Root of a Perfect Square**

When the number under a square root is factored completely, any factor that occurs twice can be taken out from under the radical.

Example 1

Simplify: $\sqrt{45}$.

Solution:

To begin, we factor 45 into the product of prime factors.

$$\begin{aligned} \sqrt{45} &= \sqrt{3 \cdot 3 \cdot 5} && \text{Factor} \\ &= \sqrt{3 \cdot 3} \cdot \sqrt{5} && \text{Multiplication property} \\ &= 3 \cdot \sqrt{5} && \text{Repeated factor property} \end{aligned}$$

The expressions $\sqrt{45}$ and $3 \cdot \sqrt{5}$ are equivalent.

Example 1 – *Solution*

cont'd

The expression $3 \cdot \sqrt{5}$ is said to be in *simplified form* because the number under the radical is as small as possible.

Example 4

Simplify: $\sqrt{48x^3}$.

Solution:

$$\sqrt{48x^3} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x}$$

$$= 2 \cdot 2 \cdot x \cdot \sqrt{3 \cdot x}$$

$$= 4x\sqrt{3x}$$

Simplifying Square Roots

FACTS FROM GEOMETRY **The Spiral of Roots**

To visualize the square roots of the positive integers, we can construct the spiral of roots. To begin, we draw two line segments, each of length 1, at right angles to each other. Then we use the Pythagorean theorem to find the length of the diagonal. Figure 1 illustrates:

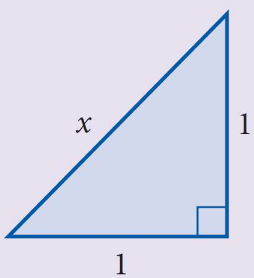

FIGURE 1

$$x = \sqrt{1^2 + 1^2}$$
$$= \sqrt{2}$$

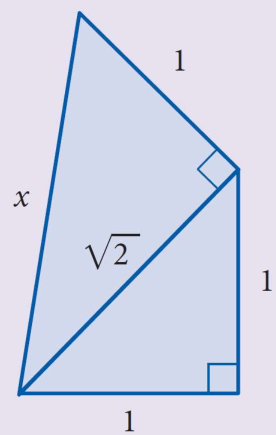

FIGURE 2

$$x = \sqrt{(\sqrt{2})^2 + 1^2}$$
$$= \sqrt{2 + 1}$$
$$= \sqrt{3}$$

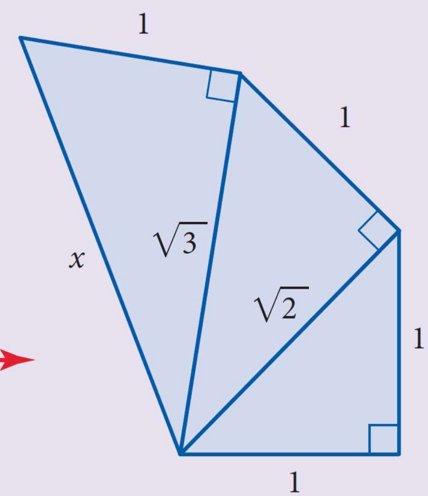

FIGURE 3

$$x = \sqrt{(\sqrt{3})^2 + 1^2}$$
$$= \sqrt{3 + 1}$$
$$= \sqrt{4}$$
$$= 2$$

Simplifying Square Roots

cont'd

Next, we construct a second triangle by connecting a line segment of length 1 to the end of the first diagonal so that the angle formed is a right angle. We find the length of the second diagonal using the Pythagorean theorem. Figure 2 illustrates this procedure. As we continue to draw new triangles by connecting line segments of length 1 to the end of each previous diagonal so that the angle formed is a right angle, the spiral of roots begins to appear (see Figure 3).