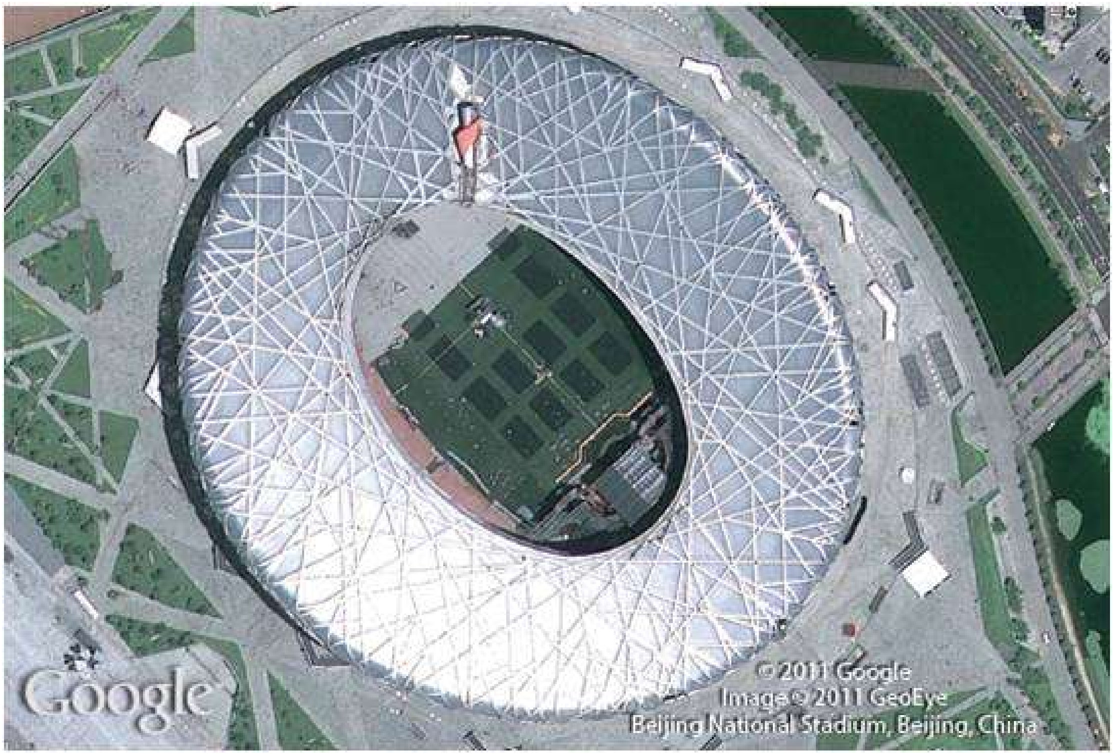
Decimals

5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SECTION 5.7

Square Roots and the Pythagorean Theorem

Objectives

A Find square roots of numbers.

B Use decimals to approximate square roots.

C Solve problems with the Pythagorean theorem.

A Square Roots

Square Roots

For example, we consider expressions like this:

$$5^2 = 5 \cdot 5 = 25$$

$$7^2 = 7 \cdot 7 = 49$$

$$x^2 = x \cdot x$$

We say that "the square of 5 is 25" and "the square of 7 is 49."

To square a number, we multiply it by itself. When we ask for the *square root* of a given number, we want to know what number we square in order to obtain the given number.

Square Roots

We say that the square root of 49 is 7, because 7 is the number we square to get 49. Likewise, the square root of 25 is 5, because $5^2 = 25$.

Remember, the symbol we use to denote square root is $\sqrt{}$, which is also called a *radical sign.*

Square Roots

Here is the precise definition of square root:

Definition

The **square root** of a positive number a, written $\sqrt{a}$, is the number we square to get a.

$$\text{If} \quad \sqrt{a} = b, \quad \text{then} \quad b^2 = a.$$

Numbers like 1, 9, and 25, whose square roots are whole numbers, are called *perfect squares.*

Example 3

Simplify: $\sqrt{\frac{25}{81}}$.

Solution:

We are looking for the number we square (multiply times itself) to get $\frac{25}{81}$.

We know that when we multiply two fractions, we multiply the numerators and multiply the denominators.

Example 3 – *Solution*

cont'd

Because 5 • 5 = 25 and 9 • 9 = 81, the square root of $\frac{25}{81}$ must be $\frac{5}{9}$.

$$\sqrt{\frac{25}{81}} = \frac{5}{9} \quad \text{because} \quad \left(\frac{5}{9}\right)^2 = \frac{5}{9} \cdot \frac{5}{9} = \frac{25}{81}$$

B Approximating Square Roots

Approximating Square Roots

So far in this section we have been concerned only with square roots of perfect squares.

The next question is, "What about square roots of numbers that are not perfect squares, like $\sqrt{7}$, for example?" We know that

$$\sqrt{4} = 2 \quad \text{and} \quad \sqrt{9} = 3$$

And because 7 is between 4 and 9, $\sqrt{7}$ should be between $\sqrt{4}$ and $\sqrt{9}$.

Approximating Square Roots

That is, $\sqrt{7}$ should be between 2 and 3.

But what is it exactly? The answer is, we cannot write it exactly in decimal or fraction form.

Because of this, it is called an *irrational number*.

We can approximate it with a decimal, but we can never write it exactly with a decimal.

Example 8

Approximate $\sqrt{301} + \sqrt{137}$ to the nearest hundredth.

Solution:

Using a calculator to approximate the square roots, we have

$$\sqrt{301} + \sqrt{137} \approx 17.349352 + 11.704700 = 29.054052$$

To the nearest hundredth, the answer is 29.05.

C The Pythagorean Theorem

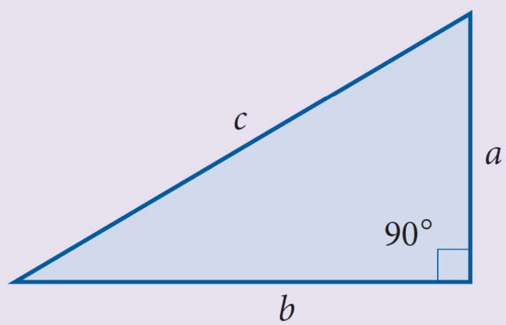
The Pythagorean Theorem

Now we'll see how square roots can help us calculate the longest side of a triangle using the *Pythagorean theorem*.

FACTS FROM GEOMETRY **Pythagorean Theorem**

A *right triangle* is a triangle that contains a 90° (or right) angle. The longest side in a right triangle is called the *hypotenuse*, and we use the letter c to denote it. The two shorter sides are denoted by the letters a and b. The Pythagorean theorem states that the hypotenuse is the square root of the sum of the squares of the two shorter sides. Here is the previous statement in symbols:

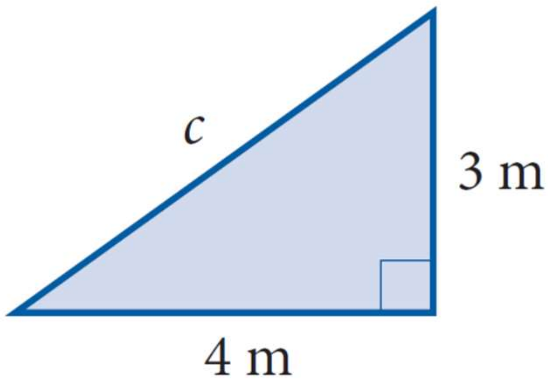
$$c = \sqrt{a^2 + b^2}$$



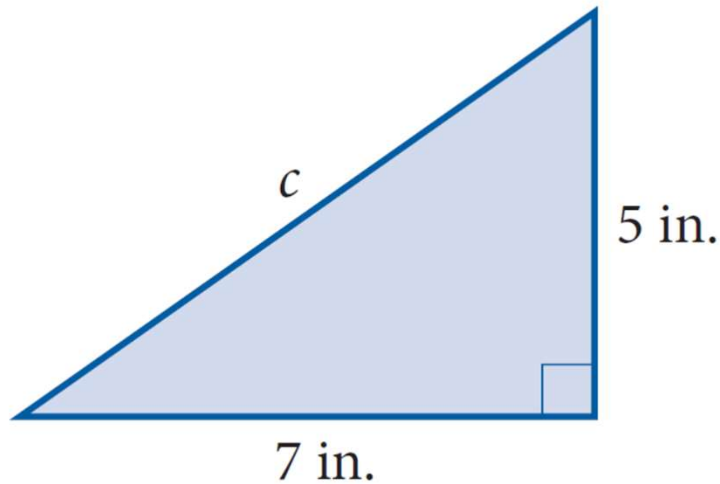
Example 10

Find the length of the hypotenuse in each right triangle.

a.



b.



Example 10 – *Solution*

We apply the formula $c = \sqrt{a^2 + b^2}$.

a. When a = 3 and b = 4,

$$c = \sqrt{3^2 + 4^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$= 5 \text{ meters}$$

Example 10 – *Solution*

cont'd

b. When *a* = 5 and *b* = 7,

$$c = \sqrt{5^2 + 7^2}$$

$$= \sqrt{25 + 49}$$

$$= \sqrt{74}$$

$$\approx 8.60 \text{ inches}$$

In part a, the solution is a whole number, whereas in part b, we must use a calculator to get 8.60 as an approximation to $\sqrt{74}$.