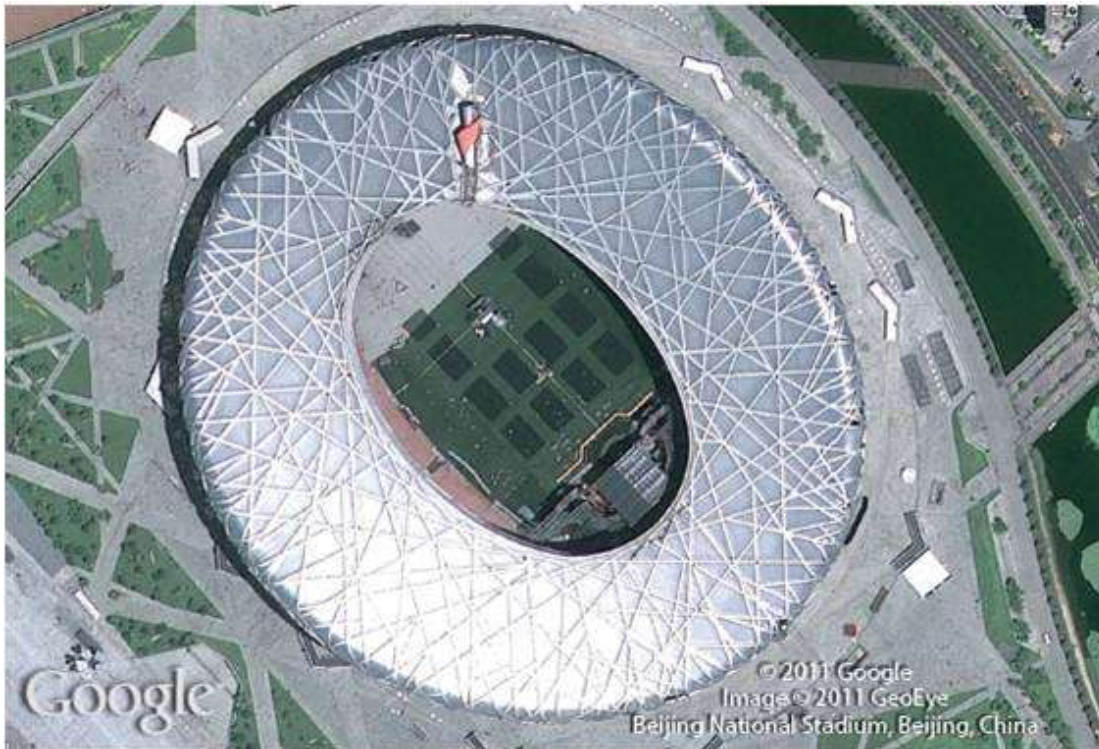


Decimals

5



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SECTION 5.5

Fractions and Decimals

Objectives

- A** Convert fractions to decimals.
- B** Convert decimals to fractions.
- C** Simplify expressions containing fractions and decimals.
- D** Solve applications involving fractions and decimals.

Fractions and Decimals

If you are shopping for clothes and a store has a sale advertising $\frac{1}{3}$ off the regular price, how much can you expect to pay for a pair of pants that normally sells for \$31.95?

If the sale price of the pants is \$22.30, have they really been marked down by $\frac{1}{3}$?

To answer questions like these, we need to know how to solve problems that involve fractions and decimals together. We begin this section by showing how to convert back and forth between fractions and decimals.



A Converting Fractions to Decimals

Converting Fractions to Decimals

You may recall that the notation we use for fractions can be interpreted as implying division.

That is, the fraction $\frac{3}{4}$ can be thought of as meaning “3 divided by 4.”

We can use this idea to convert fractions to decimals.

Example 1

Write $\frac{3}{4}$ as a decimal.

Solution:

Dividing 3 by 4, we have

$$\begin{array}{r} .75 \\ 4 \overline{)3.00} \\ \underline{28} \downarrow \\ 20 \\ \underline{20} \\ 0 \end{array}$$

The fraction $\frac{3}{4}$ is equal to the decimal 0.75.

Example 3

Write $\frac{3}{11}$ as a decimal.

Solution:

Dividing 3 by 11, we have

$$\begin{array}{r} .272727 \\ 11 \overline{) 3.000000} \\ \underline{22} \\ 80 \\ \underline{77} \\ 30 \\ \underline{22} \\ 80 \\ \underline{77} \\ 30 \\ \underline{22} \\ 80 \\ \underline{77} \\ 3 \end{array}$$

Example 3 – *Solution*

cont'd

No matter how long we continue the division, the remainder will never be 0, and the pattern will continue.

We write the decimal form of $\frac{3}{11}$; $0.\overline{27}$, etc.

$$0.\overline{27} = 0.272727 \dots$$

The dots mean "and so on."



B Converting Decimals to Fractions

Converting Decimals to Fractions

To convert decimals to fractions, we take advantage of the place values we assigned to the digits to the right of the decimal point.

Example 4

Write 0.38 as a fraction in lowest terms.

Solution:

0.38 is 38 hundredths, or

$$0.38 = \frac{38}{100}$$

$$= \frac{19}{50}$$

Divide the numerator and the denominator by 2 to reduce to lowest terms.

The decimal 0.38 is equal to the fraction $\frac{19}{50}$.

Example 4 – *Solution*

cont'd

We could check our work here by converting $\frac{19}{50}$ back to a decimal.

We do this by dividing 19 by 50. That is,

$$\begin{array}{r} .38 \\ 50 \overline{)19.00} \\ \underline{15\ 0} \downarrow \\ 4\ 00 \\ \underline{4\ 00} \\ 0 \end{array}$$

c Problems Containing Both
Fractions and Decimals

Example 7

Simplify: $\frac{19}{50}(1.32 + 0.48)$

Solution:

In Example 4, we found that $0.38 = \frac{19}{50}$.

Therefore we can rewrite the problem as

$$\frac{19}{50}(1.32 + 0.48) = 0.38(1.32 + 0.48) \quad \text{Convert all numbers to decimals.}$$

$$= 0.38(1.80) \quad \text{Add: } 1.32 + 0.48.$$

$$= 0.684 \quad \text{Multiply: } 0.38 \times 1.80.$$



Applications

Example 10

If a shirt that normally sells for \$27.99 is on sale for $\frac{1}{3}$ off, what is the sale price of the shirt?

Solution:

To find out how much the shirt is marked down, we must find $\frac{1}{3}$ of 27.99.

That is, we multiply $\frac{1}{3}$ and 27.99, which is the same as dividing 27.99 by 3.

$$\frac{1}{3}(27.99) = \frac{27.99}{3} = 9.33$$

Example 10 – *Solution*

cont'd

The shirt is marked down \$9.33. The sale price is the original price less the amount it is marked down:

$$\begin{aligned}\text{Sale price} &= 27.99 - 9.33 \\ &= 18.66\end{aligned}$$

The sale price is \$18.66.

We also could have solved this problem by simply multiplying the original price by $\frac{2}{3}$, since, if the shirt is marked $\frac{1}{3}$ off, then the sale price must be $\frac{2}{3}$ of the original price.

Example 10 – *Solution*

cont'd

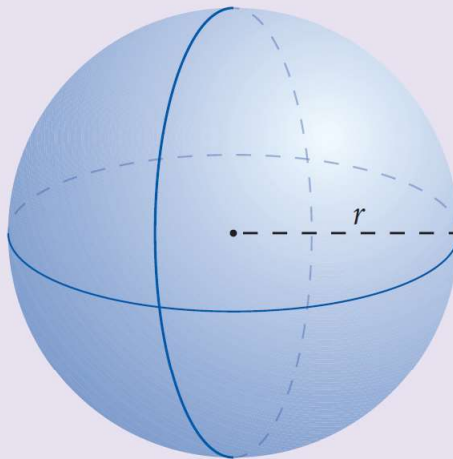
Multiplying by $\frac{2}{3}$ is the same as dividing by 3 and then multiplying by 2.

The answer would be the same.

Applications

FACTS FROM GEOMETRY The Volume of a Sphere

Figure 1 shows a sphere and the formula for its volume. Because the formula contains both the fraction $\frac{4}{3}$ and the number π , and we have been using 3.14 for π , we can think of the formula as containing both a fraction and a decimal.



$$\begin{aligned}\text{Volume} &= \frac{4}{3}\pi(\text{radius})^3 \\ &= \frac{4}{3}\pi r^3\end{aligned}$$

FIGURE 1

Example 12

Figure 2 is composed of a right circular cylinder with half a sphere on top. (A half-sphere is called a *hemisphere*.) To the nearest tenth, find the total volume enclosed by the figure.

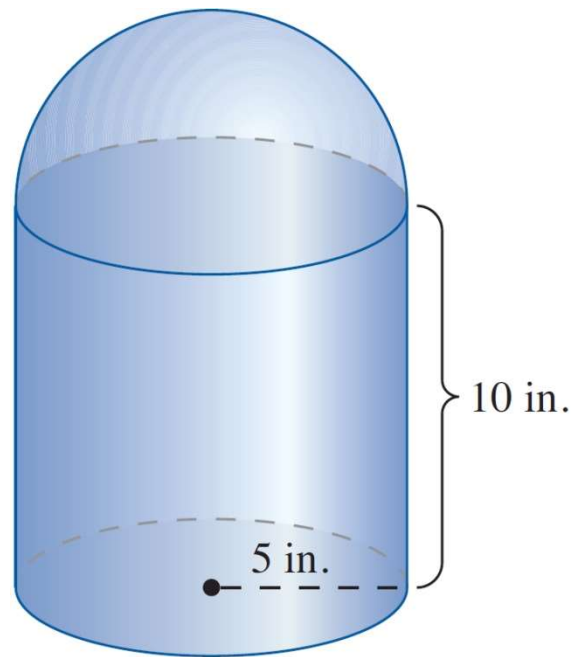


Figure 2

Example 12 – *Solution*

The total volume is found by adding the volume of the cylinder to the volume of the hemisphere.

$$V = \text{volume of cylinder} + \text{volume of hemisphere}$$

$$= \pi r^2 h + \frac{1}{2} \cdot \frac{4}{3} \pi r^3$$

$$\approx (3.14)(5)^2(10) + \frac{1}{2} \cdot \frac{4}{3}(3.14)(5)^3$$

$$\approx (3.14)(25)(10) + \frac{1}{2} \cdot \frac{4}{3}(3.14)(125)$$

$$\approx 785 + \frac{2}{3}(392.5)$$

$$\text{Multiply: } \frac{1}{2} \cdot \frac{4}{3} = \frac{4}{6} = \frac{2}{3}.$$

Example 12 – *Solution*

cont'd

$$\approx 785 + \frac{785}{3}$$

Multiply: $2(392.5) = 785$.

$$\approx 785 + 261.7$$

Divide 785 by 3, and round to the nearest tenth.

$$\approx 1,046.7 \text{ in}^3$$