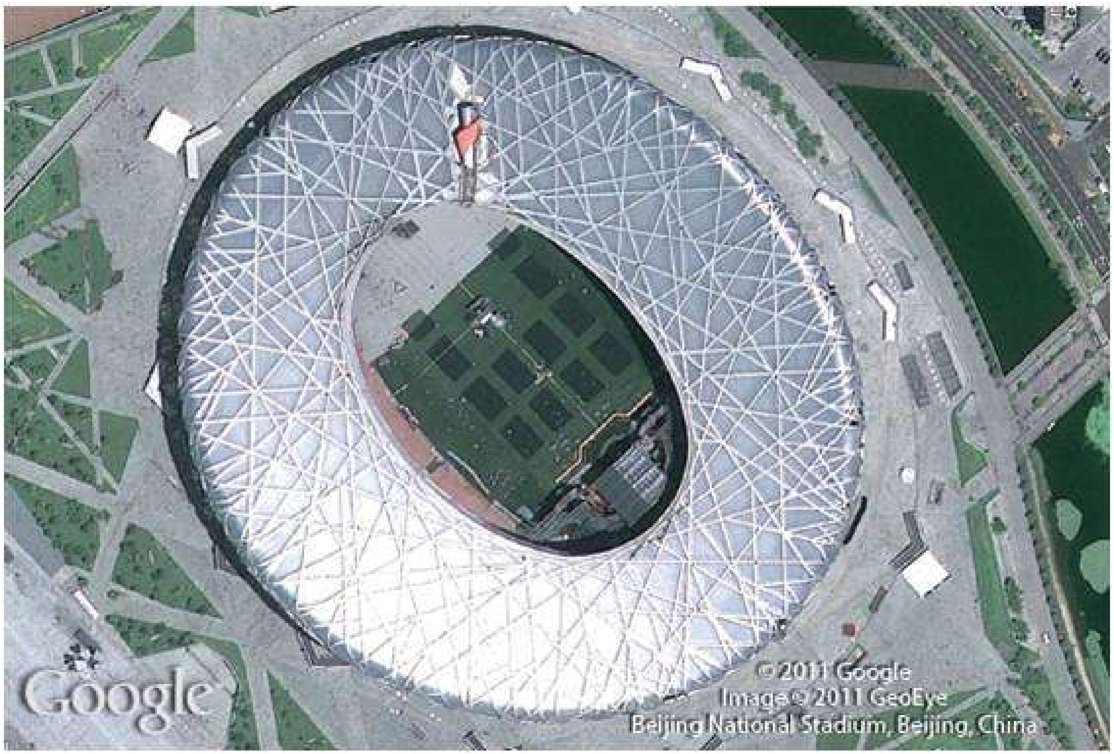
Decimals

5



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SECTION 5.3

Multiplication with Decimals; Circumference and Area of a Circle

Objectives

A Multiply decimal numbers.

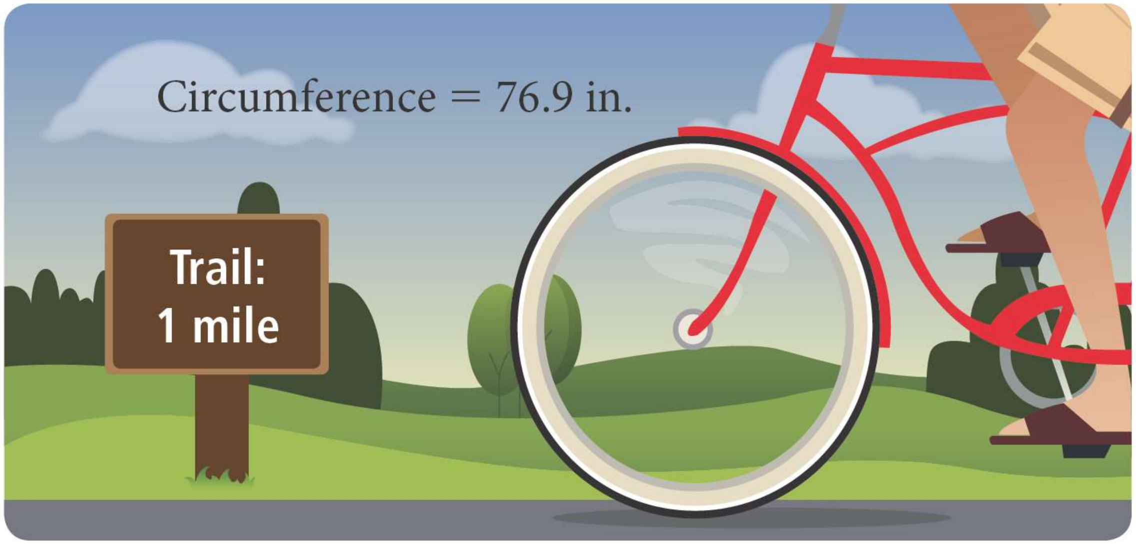
B Solve application problems involving decimals.

C Find the circumference of a circle.

The distance around a circle is called the circumference. If you know the circumference of a bicycle wheel, and you ride the bicycle for one mile, you can calculate how many times the wheel has turned through one complete revolution.

Multiplication with Decimals; Circumference and Area of a Circle

In this section, we learn how to multiply decimal numbers, and this gives us the information we need to work with circles and their circumferences.



A Multiplying with Decimals

Multiplying with Decimals

Before we introduce circumference, we need to back up and discuss multiplication with decimals.

Suppose that during a half-price sale a calendar that usually sells for \$6.42 is priced at \$3.21. Therefore it must be true that

$$\frac{1}{2} \text{ of } 6.42 \text{ is } 3.21$$

But, because $\frac{1}{2}$ can be written as 0.5 and *of* translates to *multiply*, we can write this problem again as

$$0.5 \times 6.42 = 3.21$$

Multiplying with Decimals

If we were to ignore the decimal points in this problem and simply multiply 5 and 642, the result would be 3,210.

So, multiplication with decimal numbers is similar to multiplication with whole numbers.

The difference lies in deciding where to place the decimal point in the answer.

To find out how this is done, we can use fraction notation.

Example 1

Change each decimal to a fraction and multiply:

$$0.5 \times 0.3$$

Example 1 – *Solution*

Changing each decimal to a fraction and multiplying, we have

$$0.5 \times 0.3 = \frac{5}{10} \times \frac{3}{10}$$ Change to fractions.

$$= \frac{15}{100}$$ Multiply numerators and multiply denominators.

$$= 0.15$$ Write the answer in decimal form.

The result is 0.15, which has two digits to the right of the decimal point.

Multiplying with Decimals

What we want to do now is find a shortcut that will allow us to multiply decimals without first having to change each decimal number to a fraction. Let’s look at another example.

Example 2

Change each decimal to a fraction and multiply:
0.05×0.003.

Solution:

$$0.05 \times 0.003 = \frac{5}{100} \times \frac{3}{1,000}$$ Change to fractions.

$$= \frac{15}{100,000}$$ Multiply numerators and multiply denominators.

$$= 0.00015$$ Write the answer in decimal form.

The result is 0.00015, which has a total of five digits to the right of the decimal point.

Multiplying with Decimals

Looking over these first two examples, we can see that the digits in the result are just what we would get if we simply forgot about the decimal points and multiplied; that is,
$3 \times 5 = 15$.

The decimal point in the result is placed so that the total number of digits to its right is the same as the total number of digits to the right of both decimal points in the original two numbers.

The reason this is true becomes clear when we look at the denominators after we have changed from decimals to fractions.

Multiplying with Decimals

Rule Multiplication with Decimals

To multiply two decimal numbers, follow these steps:

1. Multiply as you would if the decimal points were not there.
2. Place the decimal point in the answer so that the number of digits to its right is equal to the total number of digits to the right of the decimal points in the original two numbers in the problem.

Estimating

Example 8

Estimate the answer to each of the following products.

a. 29.4×8.2 **b.** 68.5×172 **c.** $(6.32)^2$

Solution:

a. Because 29.4 is approximately 30 and 8.2 is approximately 8, we estimate this product to be about $30 \times 8 = 240$. (If we were to multiply 29.4 and 8.2, we would find the product to be exactly 241.08.)

b. Rounding 68.5 to 70 and 172 to 170, we estimate this product to be $70 \times 170 = 11{,}900$. (The exact answer is 11,782.)

Example 8 – *Solution*

cont'd

Note here that we do not always round the numbers to the nearest whole number when making estimates.

The idea is to round to numbers that will be easy to multiply.

c. Because 6.32 is approximately 6 and $6^2 = 36$, we estimate our answer to be close to 36. (The actual answer is 39.9424.)

Combined Operations

Combined Operations

We can use the rule for order of operations to simplify expressions involving decimal numbers and addition, subtraction, and multiplication.

Example 9

Simplify: 0.05(4.2 + 0.03).

Solution:

We begin by adding inside the parentheses:

$$\begin{aligned} 0.05(4.2 + 0.03) &= 0.05(4.23) && \text{Add.} \\ &= 0.2115 && \text{Multiply.} \end{aligned}$$

Notice that we could also have used the distributive property first, and the result would be unchanged:

$$\begin{aligned} 0.05(4.2 + 0.03) &= 0.05(4.2) + 0.05(0.03) && \text{Distributive property} \\ &= 0.210 + 0.0015 && \text{Multiply.} \\ &= 0.2115 && \text{Add.} \end{aligned}$$

B Applications

Example 11

Find the area of each of the following stamps. Round to the nearest hundredth.

a.

Each side is 35.0 millimeters

b.

Length = 1.56 inches
Width = 0.99 inches

Example 11 – *Solution*

Applying our formulas for area, we have

a. $A = s^2 = (35 \text{ mm})^2 = 1{,}225 \text{ mm}^2$

b. $A = lw = (1.56 \text{ in.})(0.99 \text{ in.}) = 1.54 \text{ in}^2$

C Circumference

Circumference

Since we now have a working knowledge of decimals, we can begin our discussion on circumference. We start with two definitions.

Definition

The **radius** of a circle is the length of a straight line that stretches from the center of the circle to any point on its edge. The radius is denoted by the letter r.

Definition

The **diameter** of a circle is the distance from one side of the circle to the other, crossing through the center. The diameter is denoted by the letter d.

Circumference

FACTS FROM GEOMETRY **The Circumference of a Circle**

The *circumference* of a circle is the distance around the outside of the circle, just as the perimeter of a polygon is the distance around the outside. The circumference of a circle can be found by measuring its radius or diameter and then using the appropriate formula. In Figure 1, we can see that the diameter is twice the radius, or

$$d = 2r$$

The relationship between the circumference and the diameter or radius is not as obvious. As a matter of fact, it takes some fairly complicated mathematics to show just what the relationship between the circumference and the diameter is.

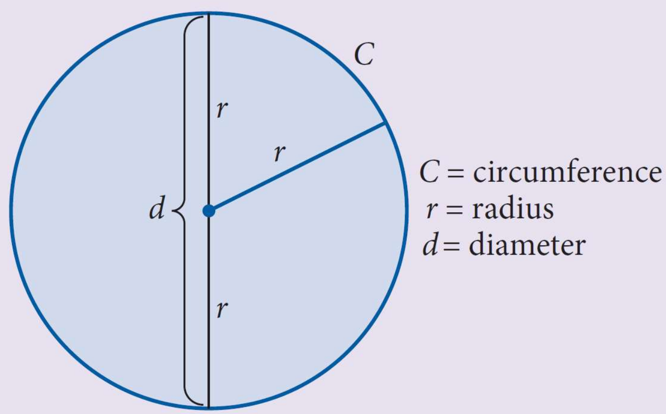


FIGURE 1

Circumference

cont'd

If you took a string and actually measured the circumference of a circle by wrapping the string around the circle and then measured the diameter of the same circle, you would find that the ratio of the circumference to the diameter, $\frac{C}{d}$, would be approximately equal to 3.14. The actual ratio of C to d in any circle is an irrational number, which is a number that can't be written in decimal form. We use the symbol π (Greek pi) to represent this ratio. In symbols, the relationship between the circumference and the diameter in any circle is

$$\frac{C}{d} = \pi$$

Knowing what we do about the relationship between division and multiplication, we can rewrite this formula as

$$C = \pi d$$

This is the formula for the circumference of a circle. For now, when we do the actual calculations, we will use the approximation 3.14 for π.

Because $d = 2r$, the same formula written in terms of the radius is

$$C = 2\pi r$$

Example 13

Find the circumference of a circle with a diameter of 5 feet.

Solution:

Substituting 5 for *d* in the formula $C = \pi d$, and using 3.14 for π, we have

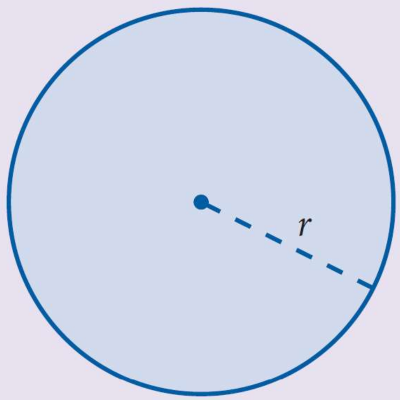
$$C \approx 3.14(5)$$

$$= 15.7 \text{ feet}$$

Circumference

FACTS FROM GEOMETRY **Other Formulas Involving π**

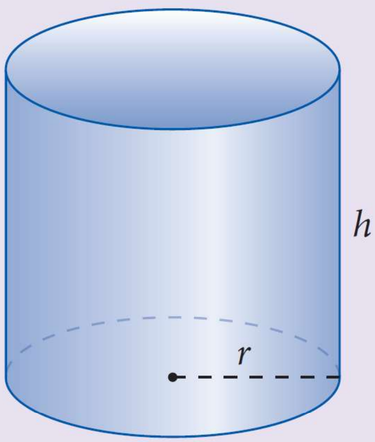
Two figures are presented here, along with some important formulas that are associated with each figure. As you can see, each of the formulas contains the number π. For now, when we do the actual calculations, we will use the approximation 3.14 for π.



Area $= \pi(\text{radius})^2$

$A = \pi r^2$

FIGURE 2 Circle



Volume $= \pi(\text{radius})^2(\text{height})$

$V = \pi r^2 h$

FIGURE 3 Right Circular Cylinder

Example 15

Find the area of a circle with a diameter of 10 feet.

Solution:

The formula for the area of a circle is $A = \pi r^2$. Because the radius r is half the diameter and the diameter is 10 feet, the radius is 5 feet.

Therefore,

$$
\begin{aligned}
A &= \pi r^2 \\
&\approx (3.14)(5)^2 \\
&= (3.14)(25) \\
&= 78.5 \text{ ft}^2
\end{aligned}
$$