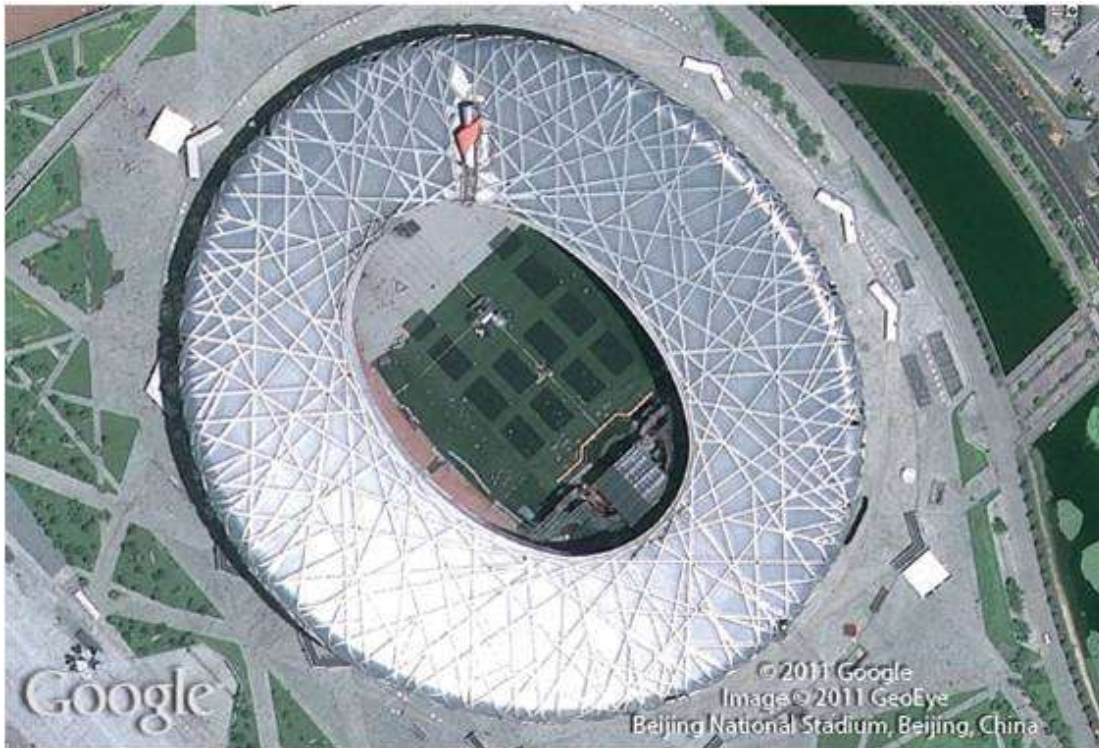


Decimals

5



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SECTION 5.1

Decimal Notation and Place Value

Objectives

- A** Understand place value for decimal numbers.
- B** Write decimal numbers in words and with digits.
- C** Convert decimals to fractions and fractions to decimals.
- D** Round a decimal number.

Decimal Notation and Place Value

In this section we will focus our attention on *decimals*. Anyone who has used money in the United States has worked with decimals already.

For example, suppose you decide to eat out at your local fast food restaurant. Once you have ordered your food, the cashier says your total bill is “6 dollars and 25 cents.” However, you see

\$6.25
↑
Decimal point



on the cash register's screen.

Decimal Notation and Place Value

The register has translated your total bill into a number that uses a decimal point.

What is interesting and useful about decimals is their relationship to fractions and to powers of ten.

The work we have done up to now—especially our work with fractions—can be used to develop the properties of decimal numbers.



A Place Value

Place Value

We have developed the idea of place value for the digits in a whole number.

We have given the name and the place value of each of the first seven columns in our number system, as follows:

Millions Column	Hundred Thousands Column	Ten Thousands Column	Thousands Column	Hundreds Column	Tens Column	Ones Column
1,000,000	100,000	10,000	1,000	100	10	1

As we move from right to left, we multiply by 10 each time.

The value of each column is 10 times the value of the column on its right, with the rightmost column being 1.

Place Value

Up until now we have always looked at place value as increasing by a factor of 10 each time we move one column to the left.

Ten Thousands	Thousands	Hundreds	Tens	Ones
10,000	1,000	100	10	1
← Multiply by 10.		← Multiply by 10.		← Multiply by 10.

Place Value

To understand the idea behind decimal numbers, we notice that moving in the opposite direction, from left to right, we *divide* by 10 each time:

Ten Thousands	Thousands	Hundreds	Tens	Ones
10,000	1,000	100	10	1
Divide by 10.		Divide by 10.		Divide by 10.

Place Value

If we keep going to the right, the next column will have to be

$$1 \div 10 = \frac{1}{10}$$

Tenths

The next one after that will be

$$\frac{1}{10} \div 10 = \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$$

Hundredths

After that, we have

$$\frac{1}{100} \div 10 = \frac{1}{100} \cdot \frac{1}{10} = \frac{1}{1,000}$$

Thousandths

Place Value

We could continue this pattern as long as we wanted. We simply divide by 10 to move one column to the right. (And remember, dividing by 10 gives the same result as multiplying by $\frac{1}{10}$.)

To show where the ones column is, we use a *decimal point* between the ones column and the tenths column.

Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths
1,000	100	10	1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$	$\frac{1}{10,000}$	$\frac{1}{100,000}$
				↑ Decimal point					

Place Value

The ones column can be thought of as the middle column, with columns larger than 1 to the left and columns smaller than 1 to the right.

The first column to the right of the ones column is the tenths column, the next column to the right is the hundredths column, the next is the thousandths column, and so on.

The decimal point is always written between the ones column and the tenths column.

We can use the place value of decimal fractions to write them in expanded form.

Example 1

Write 423.576 in expanded form.

Solution:

$$423.576 = 400 + 20 + 3 + \frac{5}{10} + \frac{7}{100} + \frac{6}{1,000}$$



B Writing Decimals with Words

Example 2

Write each number in words.

a. 0.4

b. 0.04

c. 0.004

Solution:

a. 0.4 is “four tenths.”

b. 0.04 is “four hundredths.”

c. 0.004 is “four thousandths.”

Writing Decimals with Words

When a decimal fraction contains digits to the left of the decimal point, we use the word “and” to indicate where the decimal point is when writing the number in words.

Example 3

Write each number in words.

a. 5.4

b. 5.04

c. 5.004

Solution:

a. 5.4 is “five and four tenths.”

b. 5.04 is “five and four hundredths.”

c. 5.004 is “five and four thousandths.”



c Converting Between Fractions and Decimals

Example 6

Write each number as a fraction or a mixed number. Do not reduce to lowest terms.

a. 0.004

b. 3.64

c. 25.4936

Example 6 – *Solution*

a. Because 0.004 is 4 thousandths, we write

$$0.004 = \frac{4}{1,000}$$

Three digits after the decimal point

Three zeros

b. We read the decimal part as “sixty-four hundredths”, and we can write

$$3.64 = 3\frac{64}{100}$$

Two digits after the decimal point

Two zeros

Example 6 – *Solution*

cont'd

- c. We write 25.4936 in words as “twenty-five and four thousand, nine hundred thirty-six ten thousandths”, and we have

$$25.4936 = 25 \frac{4936}{10,000}$$

Four digits after
the decimal point

Four zeros



Rounding Decimal Numbers

Example 7

Round 9,235.492 to the nearest hundred.

Solution:

The number next to the hundreds column is 3, which is less than 5.

We change all digits to the right to 0, and we can drop all digits to the right of the decimal point, so we write

9,200