# Solving Equations

# 4

**SECTION 4.9**

# Graphing Straight Lines

# Objectives

**A** Graph a line given a linear equation in two variables.

**B** Graph a vertical or horizontal line.

## A Graphing a Linear Equation in Two Variables

# Graphing a Linear Equation in Two Variables

Let's begin by working through an example that will help us develop a strategy for graphing a straight line.

# Example 1

Graph the solution set for $x + y = 4$.

Solution:

We know that there are an infinite number of solutions to this equation, and that each solution is an ordered pair of numbers.

Some of the ordered pairs that satisfy the equation $x + y = 4$ are (0, 4), (2, 2), (4, 0), and (5, –1).

# Example 1 – *Solution*

cont'd

If we plot these ordered pairs, we have the points shown in Figure 1.

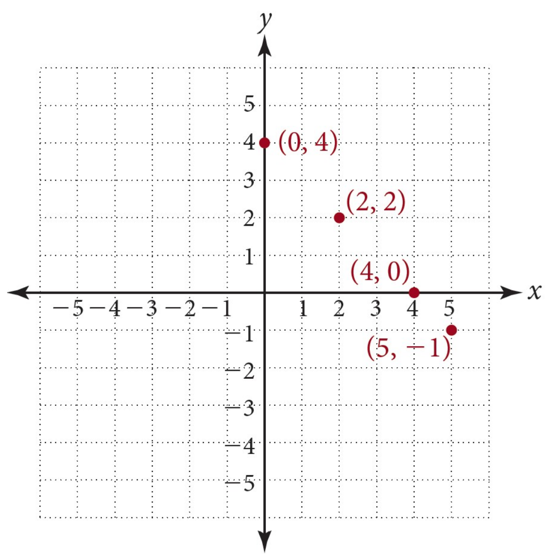


**Figure 1**

Notice that all four points lie in a straight line. If we were to find other solutions to the equation $x + y = 4$, we would find that they too would line up with the points shown in Figure 1.

# Example 1 – *Solution*

cont'd

In fact, every solution to $x + y = 4$ is a point that lies in line with the points shown in Figure 1.

Therefore, to graph the solution set to $x + y = 4$, we simply draw a line through the points in Figure 1, to obtain the graph shown in Figure 2.

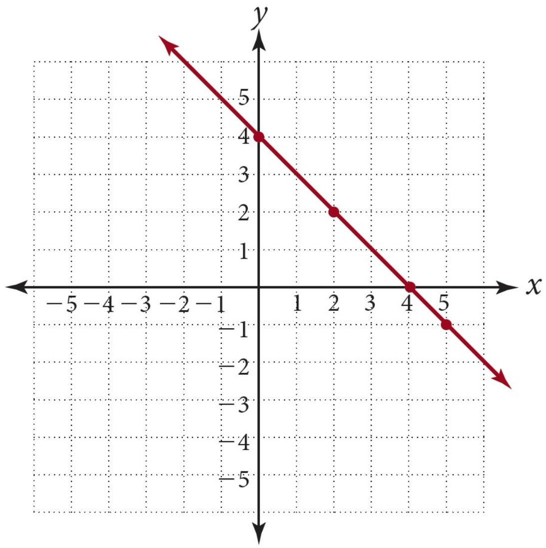


**Figure 2**

# Graphing a Linear Equation in Two Variables

We can use what we have learned in Example 1 to write the following strategy for graphing straight lines.

**Strategy** **Graphing a Straight Line**

**Step 1** Find any three ordered pairs that satisfy the equation. This is usually accomplished by substituting a number for one of the variables in the equation, and then solving for the other variable.

**Step 2** Plot the three ordered pairs found in Step 1. Actually, we need only two points to graph a straight line. The third point serves as a check. If all three points don't line up, then we have made a mistake in our work.

**Step 3** Draw a line through the three points you plotted in Step 2.

# Example 4

Graph the line $3x + 2y = 6$.

Solution:

This time, let's substitute values for both $x$ and $y$ to find the three solutions we need to draw the graph.

Let $x = 0$: $\quad 3 \cdot 0 + 2y = 6$

$$0 + 2y = 6$$

$$2y = 6$$

$$y = 3 \qquad (0, 3) \text{ is one solution.}$$

# Example 4 – *Solution*

cont'd

$$\text{Let } y = 0\text{:} \quad 3x + 2 \cdot 0 = 6$$

$$3x + 0 = 6$$

$$3x = 6$$

$$x = 2$$

(2, 0) is a second solution.

$$\text{Let } y = -3\text{:} \quad 3x + 2(-3) = 6$$

$$3x - 6 = 6$$

$$3x = 12$$

$$x = 4$$

(4, –3) is a third solution.

# Example 4 – *Solution*

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Plotting these three solutions and drawing a line through them, we have the graph shown in Figure 5.

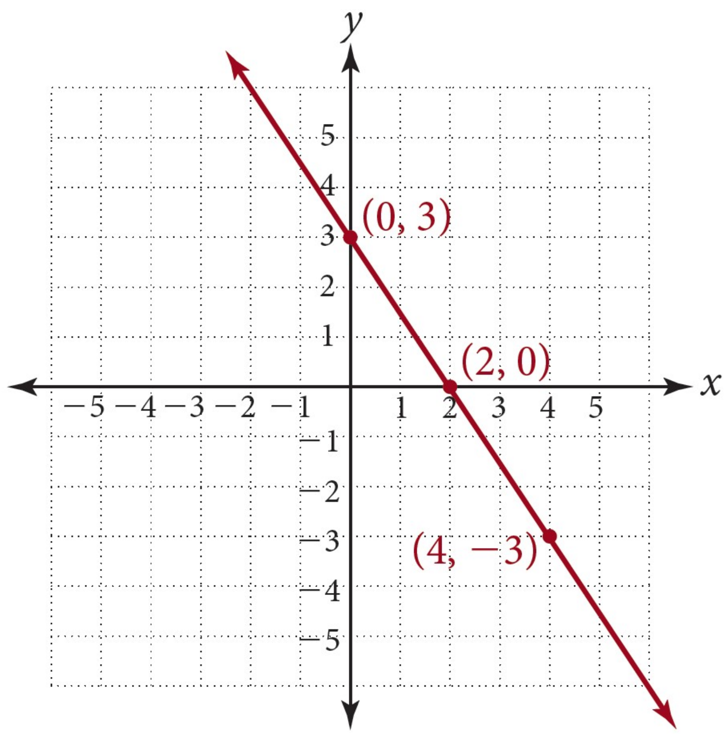


**Figure 5**

## B Horizontal and Vertical Lines

# Horizontal and Vertical Lines

We will now focus on how to recognize the equations for horizontal and vertical lines, as well as those that pass through the origin of our coordinate system.

# Example 5

Graph each of the following lines.

**a.** $y = \frac{1}{2}x$ **b.** $x = 3$ **c.** $y = 2$

Solution:

**a.** The line $y = \frac{1}{2}x$ passes through the origin because (0, 0) satisfies the equation.

To sketch the graph we need at least one more point on the line.

# Example 5 – *Solution*

cont'd

When *x* is 2, we obtain the point (2, 1), and when *x* is –4, we obtain the point (–4, –2). The graph of $y = \frac{1}{2}x$ is shown in Figure 6A.

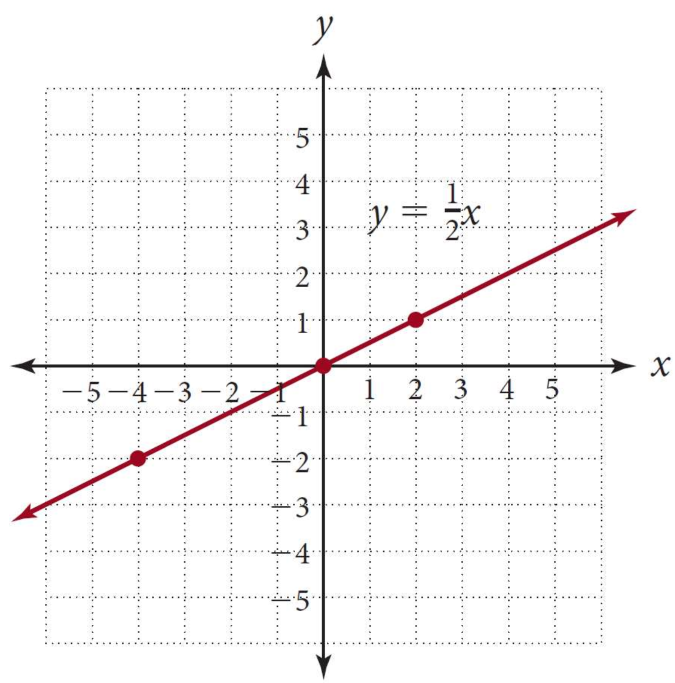


**Figure 6A**

# Example 5 – *Solution*

cont'd

**b.** The line $x = 3$ is the set of all points whose $x$-coordinate is 3. The variable $y$ does not appear in the equation, so the $y$-coordinate can be any number.

Note that we can write our equation as a linear equation in two variables by writing it as $x + 0y = 3$.

Because the product of 0 and $y$ will always be 0, $y$ can be any number.

# Example 5 – *Solution*

cont'd

The graph of $x$ = 3 is the vertical line shown in Figure 6B.

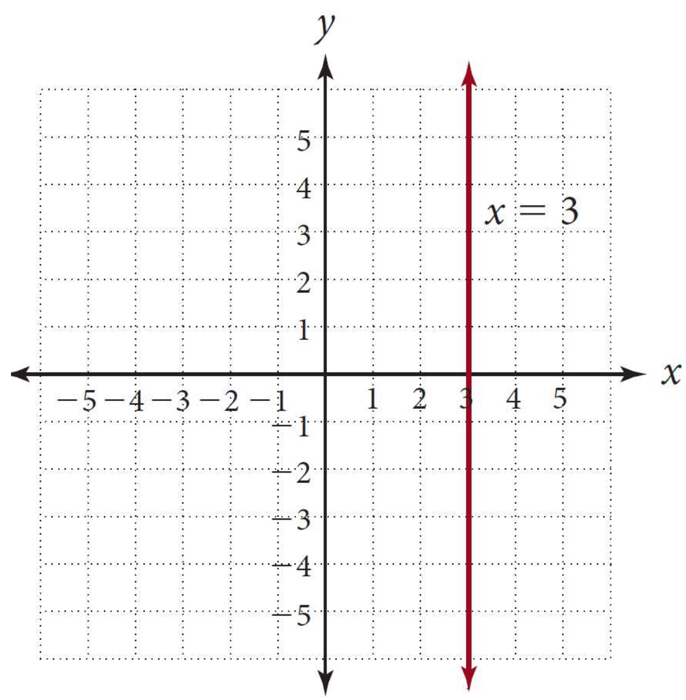


**Figure 6B**

# Example 5 – *Solution*

cont'd

**c.** The line $y = -2$ is the set of all points whose $y$-coordinate is $-2$. The variable $x$ does not appear in the equation, so the $x$-coordinate can be any number.

Again, we can write our equation as a linear equation in two variables by writing it as $0x + y = -2$.

Because the product of 0 and $x$ will always be 0, $x$ can be any number.

# Example 5 – *Solution*

cont'd

The graph of $y = -2$ is the horizontal line shown in Figure 6C.

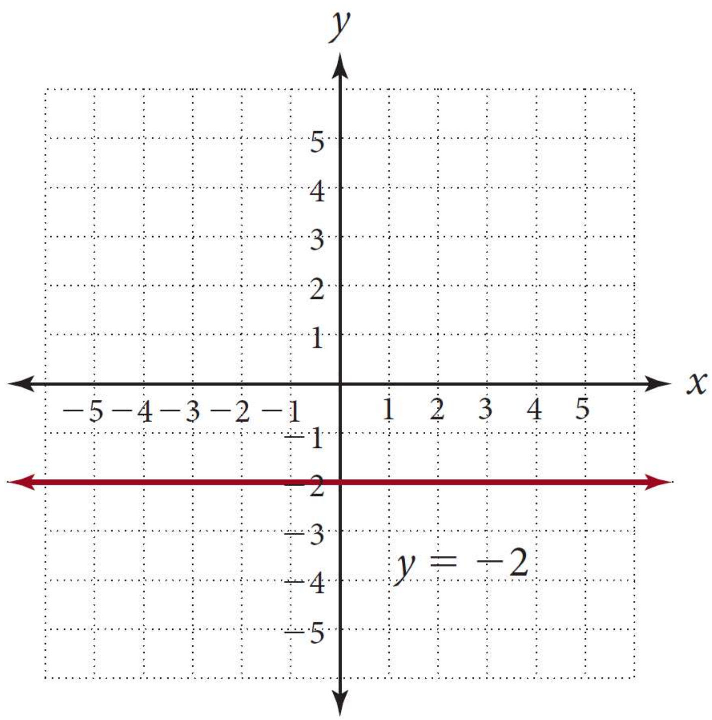


**Figure 6C**

# Horizontal and Vertical Lines

**FACTS FROM GEOMETRY** **Special Equations and Their Graphs**

For the equations below, $m$, $a$, and $b$ are numbers.

Through the Origin

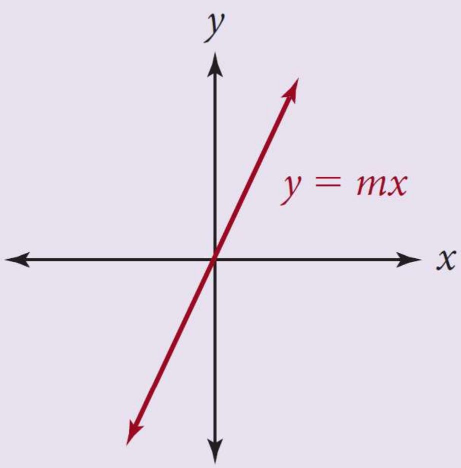


**FIGURE 7A** Any equation of the form $y = mx$ has a graph that passes through the origin.

Vertical Line

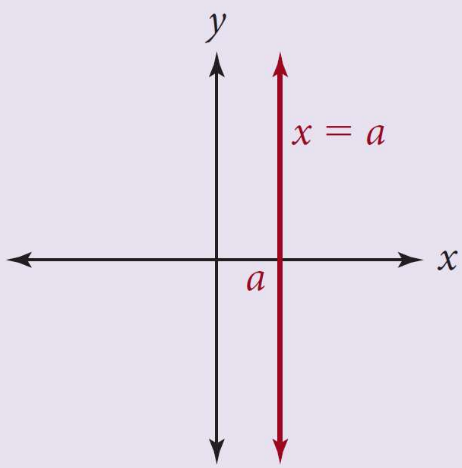


**FIGURE 7B** Any equation of the form $x = a$ has a vertical line for its graph.

Horizontal Line

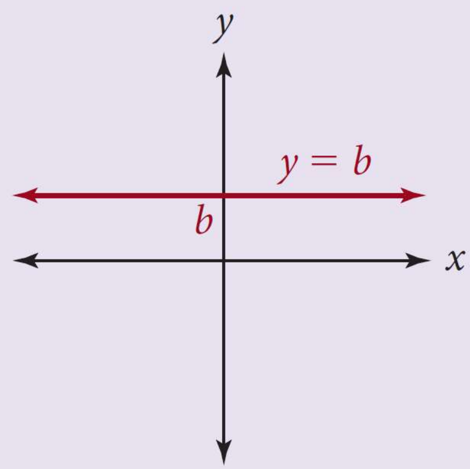


**FIGURE 7C** Any equation of the form $y = b$ has a horizontal line for its graph.