Solving Equations

4

SECTION 4.7

Paired Data and Equations in Two Variables

Objectives

A Write solutions to equations in two variables as ordered pairs.

B Fill in a table from solutions to equations in two variables.

C Evaluate ordered pairs as possible solutions to equations in two variables.

A

Solutions as Ordered Pairs

Solutions as Ordered Pairs

A solution to an equation in one variable is a single number that, when substituted for the variable in the equation, turns the equation into a true statement.

For example, $x = 5$ is a solution to the equation $2x - 4 = 6$, because replacing x with 5 turns the equation into $6 = 6$, a true statement.

Next, consider the equation $x + y = 5$. It has two variables instead of one. Therefore, a solution to $x + y = 5$ will have to consist of two numbers, one for x and one for y, that together make the equation a true statement.

Solutions as Ordered Pairs

One pair of numbers is $x = 2$ and $y = 3$, because when we substitute 2 for x and 3 for y into the equation $x + y = 5$, the result is a true statement.

To simplify our work, we write the pair of numbers
$x = 2$, $y = 3$ in the shorthand form (2, 3).

The expression (2, 3) is called an *ordered pair* of numbers.

Solutions as Ordered Pairs

Here is the formal definition:

Definition

A pair of numbers enclosed in parentheses and separated by a comma, such as (2, 3), is called an **ordered pair** of numbers. The first number in the pair is called the ***x*-coordinate** of the ordered pair, while the second number is called the ***y*-coordinate.** For the ordered pair (2, 3), the x-coordinate is 2 and the y-coordinate is 3. The general form of an ordered pair is (x, y).

In the equation $x + y = 5$, we find that (2, 3) is not the only solution. Another solution is (0, 5), because when $x = 0$ and $y = 5$, then

$$0 + 5 = 5 \quad \text{A true statement}$$

Solutions as Ordered Pairs

As you can imagine, there are many more ordered pairs that are solutions to the equation $x + y = 5$.

As a matter of fact, for any number we choose for x, there is another number we can use for y that will make the equation a true statement.

There are an infinite number of ordered pairs that are solutions to the equation $x + y = 5$.

Here is an example of how to find ordered pairs for another equation.

Example 1

Fill in the ordered pairs (0,   ), (, –2), and (3,   ) so that they are solutions to the equation $2x + 3y = 6$.

Solution:

To complete the ordered pair (0,   ), we substitute 0 for x in the equation and then solve for y.

$$\begin{aligned} \text{When} \rightarrow \quad & x = 0 \\ \text{the equation} \rightarrow \quad & 2x + 3y = 6 \\ \text{becomes} \rightarrow \quad & 2 \cdot 0 + 3y = 6 \\ & 3y = 6 \\ & y = 2 \end{aligned}$$

Example 1 – *Solution*

cont'd

Therefore, the ordered pair (0, 2) is a solution to $2x + 3y = 6$.

To complete the ordered pair (, –2), we substitute –2 for y and then solve for x.

$$\begin{aligned}
\text{When} \rightarrow \quad && y &= -2 \\
\text{the equation} \rightarrow \quad && 2x + 3y &= 6 \\
\text{becomes} \rightarrow \quad && 2x + 3(-2) &= 6 \\
&& 2x - 6 &= 6 \\
&& 2x &= 12 \\
&& x &= 6
\end{aligned}$$

Example 1 – *Solution*

cont'd

Therefore, the ordered pair (6, –2) is another solution to our equation.

Finally, to complete the ordered pair (3,   ), we substitute 3 for *x* and then solve for *y*.

The result is $y = 0$. The ordered pair (3, 0) is a third solution to our equation.

B Tables

Tables

Tables are helpful organizational tools for ordered pairs. Here is an example.

Example 2

Use the equation $5x - 2y = 20$ to complete the table below.

x	y
2	
0	
	5
	0

Solution:

Filling in the table is equivalent to completing the following ordered pairs: (2,), (0,), (, 5), and (, 0).

Example 2 – *Solution*

cont'd

We proceed as in Example 1.

When $x = 2$, we have

$$5 \cdot 2 - 2y = 20$$

$$10 - 2y = 20$$

$$-2y = 10$$

$$y = -5$$

When $x = 0$, we have

$$5 \cdot 0 - 2y = 20$$

$$0 - 2y = 20$$

$$-2y = 20$$

$$y = -10$$

Example 2 – *Solution*

cont'd

When $y = 5$, we have

$$5x - 2 \cdot 5 = 20$$
$$5x - 10 = 20$$
$$5x = 30$$
$$x = 6$$

When $y = 0$, we have

$$5x - 2 \cdot 0 = 20$$
$$5x - 0 = 20$$
$$5x = 20$$
$$x = 4$$

Using these results, we complete our table.

x	y
2	−5
0	−10
6	5
4	0

C Evaluating Ordered Pairs

Evaluating Ordered Pairs

The next example shows how we can determine if an ordered pair is a solution to an equation.

Example 3

Which of the ordered pairs (1, 5) and (2, 4) are solutions to the equation $y = 3x + 2$?

Solution:

If an ordered pair is a solution to an equation, then it must yield a true statement when the coordinates of the ordered pair are substituted for x and y in the equation.

First, we try (1, 5) in the equation $y = 3x + 2$:

$$5 = 3 \cdot 1 + 2$$

$$5 = 3 + 2$$

$$5 = 5 \qquad \text{A true statement}$$

Example 3 – *Solution*

cont'd

Next, we try (2, 4) in the equation:

$$4 = 3 \cdot 2 + 2$$

$$4 = 6 + 2$$

$$4 = 8 \qquad \text{A false statement}$$

The ordered pair (1, 5) is a solution to the equation $y = 3x + 2$, but (2, 4) is not a solution to the equation.