Solving Equations

4

SECTION 4.6

Evaluating Formulas

Objectives

A Solve a formula for a given variable.

B Solve problems using the rate equation.

Evaluating Formulas

In mathematics, a *formula* is an equation that contains more than one variable.

The equation $P = 2w + 2l$ is an example of a formula.

This formula tells us the relationship between the perimeter P of a rectangle, its length l, and its width w.

Evaluating Formulas

There are many formulas with which you may be familiar already.

Perhaps you have used the formula $d = r \cdot t$ to find out how far you would go if you traveled at 50 miles an hour for 3 hours.

If you take a chemistry class while you are in college, you will certainly use the formula that gives the relationship between the two temperature scales, Fahrenheit and Celsius.

$$F = \frac{9}{5}C + 32$$

A Formulas

Formulas

Definition

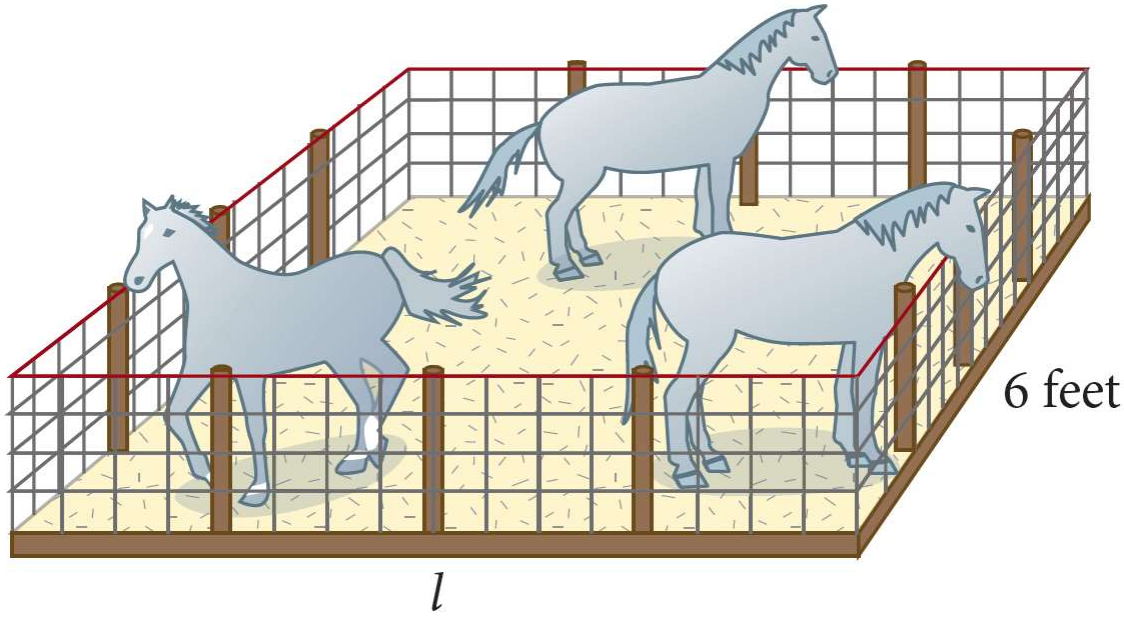
In mathematics, a **formula** is an equation with more than one variable.

Although there are many kinds of problems we can work using formulas, we will limit ourselves to those that require only substitutions.

The example that follows illustrates this type of problem.

Example 1

The perimeter P of a rectangular livestock pen is 40 feet. If the width w is 6 feet, find the length.



Example 1 – *Solution*

First we substitute 40 for P and 6 for w in the formula $P = 2l + 2w$.

Then we solve for l:

When $\rightarrow$ $P = 40$ and $w = 6$

the formula $\rightarrow$ $P = 2l + 2w$

becomes $\rightarrow$ $40 = 2l + 2(6)$

$40 = 2l + 12$ Multiply 2 and 6.

$28 = 2l$ Add -12 to each side.

$14 = l$ Multiply each side by $\frac{1}{2}$.

Example 1 – *Solution*

cont'd

To summarize our results, if a rectangular pen has a perimeter of 40 feet and a width of 6 feet, then the length must be 14 feet.

B Rate Equation

Rate Equation

Now we will look at some problems that use what is called the *rate equation*.

You use this equation on an intuitive level when you are estimating how long it will take you to drive long distances.

For example, if you drive at 50 miles per hour for 2 hours, you will travel 100 miles.

Here is the rate equation:

Distance = rate • time, or $d = r \cdot t$

Rate Equation

The rate equation has two equivalent forms, one of which is obtained by solving for *r*, while the other is obtained by solving for *t*.

Here they are:

$$r = \frac{d}{t} \text{ and } t = \frac{d}{r}$$

The rate in this equation is also referred to as *average speed*.

Example 5

At 1 p.m., Jordan leaves her house and drives at an average speed of 50 miles per hour to her sister's house. She arrives at 4 p.m.

a. How many hours was the drive to her sister's house?

b. How many miles from her sister does Jordan live?

Solution:

a. If she left at 1:00 p.m. and arrived at 4:00 p.m., we simply subtract 1 from 4 for an answer of 3 hours.

Example 5 – *Solution*

cont'd

b. We are asked to find a distance in miles given a rate of 50 miles per hour and a time of 3 hours.

We will use the rate equation,

$d = r \cdot t$, to solve this. We have

$$d = 50 \text{ miles per hour} \cdot 3 \text{ hours}$$

$$d = 50(3)$$

$$d = 150 \text{ miles}$$

Rate Equation

Notice in Example 5, that we were asked to find a distance in miles, so our answer has a unit of miles.

When we are asked to find a time, our answer will include a unit of time, like days, hours, minutes, or seconds.

When we are asked to find a rate, our answer will include units of rate, like miles per hour, feet per second, problems per minute, and so on.

Rate Equation

FACTS FROM GEOMETRY

Earlier we defined complementary angles as angles that add to 90°. That is, if x and y are complementary angles, then

$$x + y = 90°$$

If we solve this formula for y, we obtain a formula equivalent to our original formula:

$$y = 90° - x$$

Because y is the complement of x, we can generalize by saying that the complement of angle x is the angle $90° - x$. By a similar reasoning process, we can say that the supplement of angle x is the angle $180° - x$. To summarize, if x is an angle, then

the complement of x is $90° - x$, and
the supplement of x is $180° - x$.

If you go on to take a trigonometry class, you will see these formulas again.

Example 6

Find the complement and the supplement of 25°

Solution:

We can use the formulas above with $x = 25°$.

The complement of 25° is 90° – 25° = 65°.

The supplement of 25° is 180° – 25° = 155°.