# Solving Equations

# 4

SECTION 4.5

# Applications

# Objective

**A** Use the Blueprint for Problem Solving to solve a variety of application problems.

# A The Blueprint for Problem Solving

# The Blueprint for Problem Solving

To begin this section, we list the steps used in solving application problems. We call this strategy the *Blueprint for Problem Solving*.

It is an outline that will overlay the solution process we use on all application problems.

# The Blueprint for Problem Solving

**Strategy** **Blueprint for Problem Solving**

**Step 1** **Read** the problem, and then mentally **list** the items that are known and the items that are unknown.

**Step 2** **Assign a variable** to one of the unknown items. (In most cases, this will amount to letting $x$ equal the item that is asked for in the problem.) Then **translate** the other **information** in the problem to expressions involving the variable.

**Step 3** **Reread** the problem, and then **write an equation**, using the items and variables listed in Steps 1 and 2, that describes the situation.

**Step 4** **Solve the equation** found in Step 3.

**Step 5** **Write** your **answer** using a complete sentence.

**Step 6** **Reread** the problem, and **check** your solution with the original words in the problem.

# The Blueprint for Problem Solving

There are a number of substeps within each of the steps in our blueprint.

For instance, with Steps 1 and 2 it is always a good idea to draw a diagram or picture if it helps you to visualize the relationship between the items in the problem.

# Number Problems

# Example 2

If 5 is added to the sum of twice a number and three times the number, the result is 25. Find the number.

Solution:

***Step 1:*** *Read and list*.

*Known items:* The numbers 5 and 25, twice a number, and three times a number

*Unknown item:* The number in question

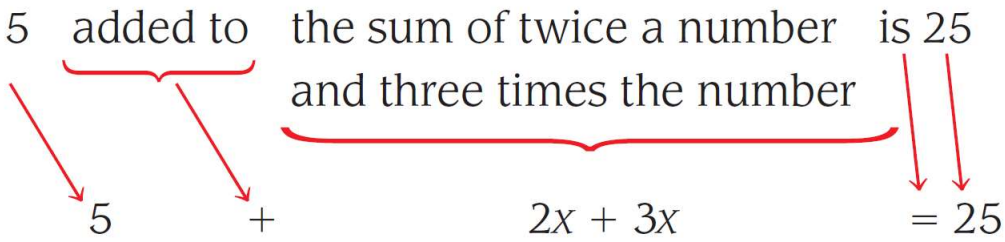
***Step 2:*** *Assign a variable and translate the information*.

# Example 2 – *Solution*

cont'd

Let $x$ = the number asked for in the problem. Then "The sum of twice a number and three times the number" translates to $2x + 3x$.

***Step 3:*** *Reread and write an equation*.




***Step 4:*** *Solve the equation*.

$$5 + 2x + 3x = 25$$

# Example 2 – *Solution*

cont'd

$5x + 5 = 25$ Simplify the left side.

$5x + 5 + \mathbf{(-5)} = 25 + \mathbf{(-5)}$ Add −5 to both sides.

$5x = 20$

$\frac{5x}{\mathbf{5}} = \frac{20}{\mathbf{5}}$ Divide by 5.

$x = 4$

***Step 5:*** *Write your answer*. The number is 4.

# Example 2 – *Solution*

cont'd

***Step 6:*** *Reread and check*.

Twice **4** is 8, and three times **4** is 12. Their sum is 8 + 12 = 20. Five added to this is 25. Therefore, 5 added to the sum of twice **4** and three times **4** is 25.

# Geometry Problems

# Example 3

The length of a rectangle is three times the width. The perimeter is 72 centimeters. Find the width and the length.

Solution:

***Step 1:*** *Read and list*.

*Known items*: The length is three times the width.
The perimeter is 72 centimeters.

*Unknown items*: The length and the width

# Example 3 – *Solution*

cont'd

***Step 2:*** *Assign a variable, and translate the information*.

We let $x$ = the width. Because the length is three times the width, the length must be $3x$. A picture will help.

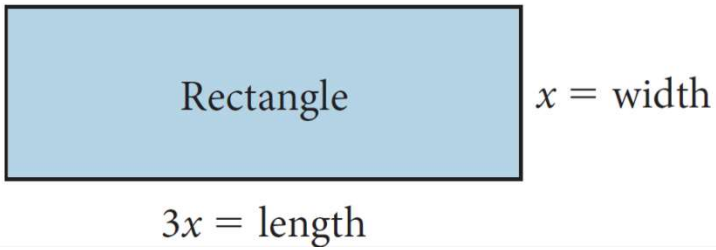



**Figure 1**

# Example 3 – *Solution*

cont'd

***Step 3:*** *Reread and write an equation.*

Because the perimeter is the sum of the sides, it must be

$x + x + 3x + 3x$ (the sum of the four sides).

But the perimeter is also given as 72 centimeters.

Hence,

$$x + x + 3x + 3x = 72$$

# Example 3 – *Solution*

cont'd

***Step 4:*** *Solve the equation*.

$$x + x + 3x + 3x = 72$$

$$8x = 72$$

$$x = 9$$

***Step 5:*** *Write your answer*.

The width, $x$, is 9 centimeters. The length, $3x$, must be 27 centimeters.

# Example 3 – *Solution*

cont'd

***Step 6:*** *Reread and check.*

From the diagram below, we see that these solutions check.

Length $= 3 \times$ width

$27 = 3 \cdot 9$

Perimeter is 72.

$9 + 9 + 27 + 27 = 72$

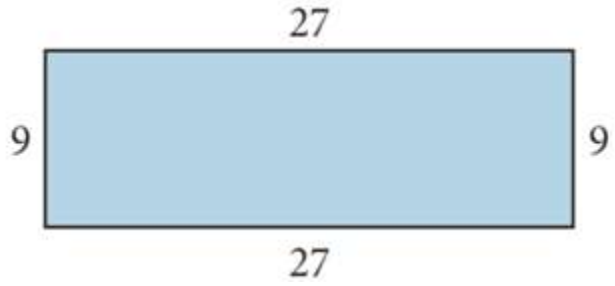



**Figure 2**

# Geometry Problems

**FACTS FROM GEOMETRY** **Labeling Triangles and the Sum of the Angles in a Triangle**

Recall that one way to label the important parts of a triangle is to label the vertices with capital letters and the sides with small letters, as shown in Figure 3.

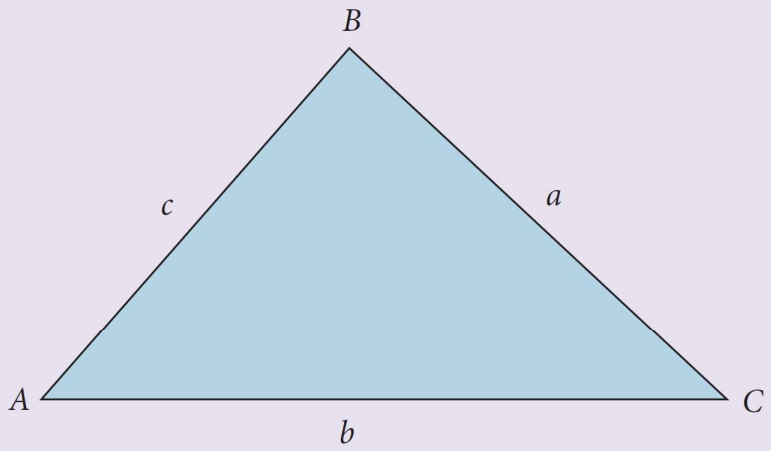



**FIGURE 3**

In Figure 3, notice that side $a$ is opposite vertex $A$, side $b$ is opposite vertex $B$, and side $c$ is opposite vertex $C$. Also, because each vertex is the vertex of one of the angles of the triangle, we refer to the three interior angles as $A$, $B$, and $C$.

In any triangle, the sum of the interior angles is 180°. For the triangle shown in Figure 3, the relationship is written

$$A + B + C = 180^\circ$$

# Geometry Problems

We can apply the Blueprint for Problem Solving to the forthcoming triangle problem.

# Example 4

The angles in a triangle are such that one angle is twice the smallest angle, while the third angle is three times as large as the smallest angle.
Find the measure of all three angles.

Solution:

***Step 1:*** *Read and list*.

*Known items*: The sum of all three angles is 180°; one angle is twice the smallest angle; and the largest angle is three times the smallest angle.

*Unknown items*: The measure of each angle

# Example 4 – *Solution*

cont'd

***Step 2:*** *Assign a variable and translate information.*

Let $x$ be the smallest angle, then $2x$ will be the measure of another angle, and $3x$ will be the measure of the largest angle.

***Step 3:*** *Reread and write an equation.*

When working with geometric objects, drawing a generic diagram will help us visualize what it is that we are asked to find. In Figure 4, we draw a triangle with angles $A$, $B$, and $C$.

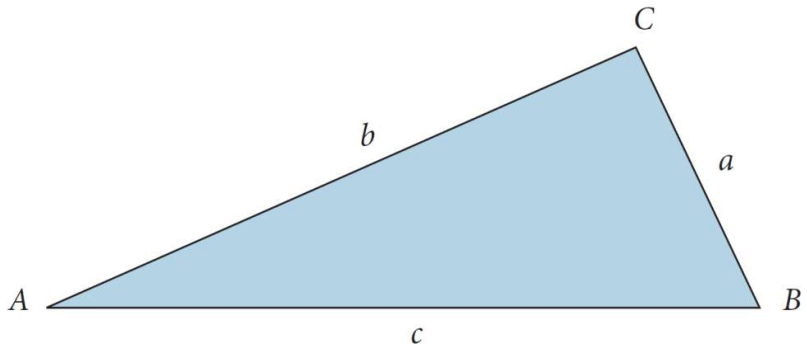



**Figure 4**

# Example 4 – *Solution*

cont'd

We can let the value of $A = x$, the value of $B = 2x$, and the value of $C = 3x$. We know that the sum of angles $A$, $B$, and $C$ will be 180°, so our equation becomes

$$x + 2x + 3x = 180°$$

***Step 4:*** *Solve the equation*.

$$x + 2x + 3x = 180°$$

$$6x = 180°$$

$$x = 30°$$

# Example 4 – *Solution*

cont'd

***Step 5:*** *Write the answer.*

The smallest angle $A$ measures 30°.

Angle $B$ measures $2x$, or $2(30°) = 60°$.

Angle $C$ measures $3x$, or $3(30°) = 90°$.

***Step 6:*** *Reread and check.*

The angles must add to 180°:

$$A + B + C = 180°$$

$$30° + 60° + 90° = 180°$$

$$180° = 180°$$

Our answers check.

# Age Problem

# Example 5

Jo Ann is 22 years older than her daughter Stacey. In six years, the sum of their ages will be 42. How old are they now?

Solution:

***Step 1:*** *Read and list*.

*Known items*: Jo Ann is 22 years older than Stacey. Six years from now their ages will add to 42.

*Unknown items*: Their ages now

# Example 5 – *Solution*

cont'd

***Step 2:*** *Assign a variable and translate the information*.

Let $x$ = Stacey's age now. Because Jo Ann is 22 years older than Stacey, her age is $x + 22$.

***Step 3:*** *Reread and write an equation*.

As an aid in writing the equation, we use the following table:

| | Now | In Six years |
|---|---|---|
| Stacey | $x$ | $x + 6$ |
| Jo Ann | $x + 22$ | $x + 28$ |

Their ages in six years will be their ages now plus 6.

# Example 5 – *Solution*

cont'd

Because the sum of their ages six years from now is 42, we write the equation as

$$(x + 6) + (x + 28) = 42$$

↑ Stacey's age in 6 years

↑ Jo Ann's age in 6 years

***Step 4:*** *Solve the equation*.

$$x + 6 + x + 28 = 42$$

$$2x + 34 = 42$$

$$2x = 8$$

$$x = 4$$

# Example 5 – *Solution*

cont'd

***Step 5:*** *Write your answer*.

Stacey is now 4 years old, and Jo Ann is

4 + 22 = 26 years old.

***Step 6:*** *Reread and check*.

To check, we see that in six years, Stacey will be 10, and Jo Ann will be 32.

The sum of 10 and 32 is 42, which checks.

# Car Rental Problem

# Example 6

A car rental company charges $11 per day and 16 cents per mile for their cars. If a car was rented for 1 day and the charge was $25.40, how many miles was the car driven?

Solution:

***Step 1:*** *Read and list*.

*Known items*: Charges are $11 per day and 16 cents per mile. Car is rented for 1 day. Total charge is $25.40.

*Unknown items*: How many miles the car was driven

# Example 6 – *Solution*

cont'd

***Step 2:*** *Assign a variable and translate information*.

If we let $x$ = the number of miles driven, then the charge for the number of miles driven will be $0.16x$, the cost per mile times the number of miles.

***Step 3:*** *Reread and write an equation*.

To find the total cost to rent the car, we add 11 to $0.16x$. Here is the equation that describes the situation:

$$\underbrace{\text{\$11 per day}}_{11} + \underbrace{\text{16 cents per mile}}_{0.16x} = \underbrace{\text{Total cost}}_{25.40}$$

# Example 6 – *Solution*

cont'd

***Step 4:*** *Solve the equation*.

To solve the equation, we add –11 to each side and then divide each side by 0.16.

$$11 + \mathbf{(-11)} + 0.16x = 25.40 + \mathbf{(-11)}$$ Add –11 to each side.

$$0.16x = 14.40$$

$$\frac{0.16x}{\mathbf{0.16}} = \frac{14.40}{\mathbf{0.16}}$$ Divide each side by 0.16.

$$x = 90$$ $14.40 \div 0.16 = 90$

# Example 6 – *Solution*

cont'd

***Step 5:*** *Write the answer*.

The car was driven 90 miles.

***Step 6:*** *Reread and check*.

The charge for 1 day is $11. The 90 miles adds 90($0.16) = $14.40 to the 1-day charge.

The total is $11 + $14.40 = $25.40, which checks with the total charge given in the problem.

# Coin Problem

# Example 7

Diane has $1.60 in dimes and nickels. If she has 7 more dimes than nickels, how many of each coin does she have?

Solution:

***Step 1:*** *Read and list*.

*Known items*: We have dimes and nickels. There are 7 more dimes than nickels, and the total value of the coins is $1.60.

*Unknown items*: How many of each type of coin Diane has

# Example 7 – *Solution*

cont'd

***Step 2:*** *Assign a variable and translate information*.

If we let $x$ = the number of nickels, then the number of dimes must be $x + 7$, because Diane has 7 more dimes than nickels.

Because each nickel is worth 5 cents, the amount of money she has in nickels is $0.05x$. Similarly, because each dime is worth 10 cents, the amount of money she has in dimes is $0.10(x + 7)$.
Here is a table that summarizes what we have so far:

| | Nickels | Dimes |
|---|---|---|
| Number of | $x$ | $x + 7$ |
| Value of | $0.05x$ | $0.10(x + 7)$ |

# Example 7 – *Solution*

cont'd

***Step 3:*** *Reread and write an equation*.

Because the total value of all the coins is \$1.60, the equation that describes this situation is

$$\underbrace{\text{Amount of money in nickels}}_{0.05x} + \underbrace{\text{Amount of money in dimes}}_{0.10(x+7)} = \underbrace{\text{Total amount of money}}_{1.60}$$

***Step 4:*** *Solve the equation*.

Let's show the essential steps in the solution.

$$0.05x + 0.10x + 0.70 = 1.60$$ Distributive property

# Example 7 – *Solution*

cont'd

$0.15x + 0.70 = 1.60$ Add $0.05x$ and $0.10x$ to get $0.15x$.

$0.15x = 0.90$ Add $-0.70$ to each side.

$x = 6$ Divide each side by 0.15.

# Example 7 – *Solution*

cont'd

***Step 5:*** *Write the answer*.

Because $x = 6$, Diane has 6 nickels. To find the number of dimes, we add 7 to the number of nickels (she has 7 more dimes than nickels).

The number of dimes is $6 + 7 = 13$.

***Step 6:*** *Reread and check*.

Here is a check of our results.

6 nickels are worth $6(\$0.05) = \$0.30$

13 dimes are worth $13(\$0.10) = \$1.30$

The total value is $\$1.60$