Solving Equations

4

SECTION 4.4

Linear Equations in One Variable

Objectives

A Solve linear equations with one variable.

B Solve linear equations involving fractions and decimals.

Linear Equations in One Variable

The Rhind Papyrus is an ancient Egyptian document, created around 1650 B.C., that contains some mathematical riddles.

One problem on the Rhind Papyrus asked the reader to find a quantity such that when it is added to one-fourth of itself the sum is 15.

The equation that describes this situation is

$$x + \frac{1}{4}x = 15$$

As you can see, this equation contains a fraction.

Linear Equations in One Variable

One of the topics we will discuss in this section is how to solve equations that contain fractions.

In this chapter we have been solving what are called *linear equations in one variable*.

They are equations that contain only one variable, and that variable is always raised to the first power and never appears in a denominator.

Linear Equations in One Variable

Here are some examples of linear equations in one variable:

$$3x + 2 = 17, \quad 7a + 4 = 3a - 2, \quad 2(3y - 5) = 6$$

Because of the work we have done in the earlier sections, we are now able to solve any linear equation in one variable.

A Solving Linear Equations with One Variable

Solving Linear Equations with One Variable

The steps outlined here can be used as a guide to solving these equations.

Strategy **Solving a Linear Equation with One Variable**

Step 1 Simplify each side of the equation as much as possible. This step is done using the commutative, associative, and distributive properties.

Step 2 Use the addition property of equality to get all variable terms on one side of the equation and all constant terms on the other, and then combine like terms. A *variable term* is any term that contains the variable. A *constant term* is any term that contains only a number.

Step 3 Use the multiplication property of equality to get the variable by itself on one side of the equation.

Step 4 Check your solution in the original equation if you think it is necessary.

Example 1

Solve: $3(x + 2) = -9$.

Solution:

We begin by applying the distributive property to the left side.

Step 1

$$3(x + 2) = -9$$
$$3x + 6 = -9 \qquad \text{Distributive property}$$

Step 2

$$3x + 6 + \mathbf{(-6)} = -9 + \mathbf{(-6)}$$
$$3x = -15 \qquad \text{Add } -6 \text{ to both sides.}$$

Example 1 – *Solution*

cont'd

Step 3

$$\frac{3x}{\mathbf{3}} = \frac{-15}{\mathbf{3}}$$

$$x = -5$$

Divide both sides by 3 .

Solving Linear Equations with One Variable

This general method of solving linear equations involves using the two properties.

We can add any number to both sides of an equation or multiply (or divide) both sides by the same nonzero number and always be sure we have not changed the solution to the equation.

The equations may change in form, but the solution to the equation stays the same.

Looking back to Example 1, we can see that each equation looks a little different from the preceding one.

Solving Linear Equations with One Variable

What is interesting, and useful, is that each of the equations says the same thing about x.

They all say that x is -5.

The last equation, of course, is the easiest to read.

That is why our goal is to end up with x isolated on one side of the equation.

B Equations Involving Fractions

Equations Involving Fractions

We will now solve some equations that involve fractions.

Because integers are usually easier to work with than fractions, we will begin each problem by clearing the equation we are trying to solve of all fractions.

To do this, we will use the multiplication property of equality to multiply each side of the equation by the LCD for all fractions appearing in the equation.

Example 5

Solve the equation $\frac{x}{2} + \frac{x}{6} = 8$.

Solution:

The LCD for the fractions $\frac{x}{2}$ and $\frac{x}{6}$ is 6.

It has the property that both 2 and 6 divide it evenly.

Therefore, if we multiply both sides of the equation by 6, we will be left with an equation that does not involve fractions.

$$\mathbf{6}\left(\frac{x}{2} + \frac{x}{6}\right) = \mathbf{6}(8)$$ Multiply each side by 6.

$$6\left(\frac{x}{2}\right) + 6\left(\frac{x}{6}\right) = 6(8)$$ Apply the distributive property.

Example 5 – *Solution*

cont'd

$$3x + x = 48$$

$$4x = 48 \qquad \text{Combine similar terms.}$$

$$x = 12 \qquad \text{Divide each side by 4.}$$

We could check our solution by substituting 12 for *x* in the original equation.

If we do so, the result is a true statement.

The solution is 12.

Equations Involving Fractions

As you can see from Example 5, the most important step in solving an equation that involves fractions is the first step.

In that first step, we multiply both sides of the equation by the LCD for all the fractions in the equation.

After we have done so, the equation is clear of fractions because the LCD has the property that all the denominators divide it evenly.

Equations Containing Decimals

Example 8

Solve: $\frac{1}{2}x - 3.78 = 2.52$

Solution:

We begin by adding 3.78 to each side of the equation.

Then we multiply each side by 2.

$$\frac{1}{2}x - 3.78 = 2.52$$

$$\frac{1}{2}x - 3.78 + \mathbf{3.78} = 2.52 + \mathbf{3.78} \qquad \text{Add 3.78 to each side.}$$

Example 8 – *Solution*

cont'd

$$\frac{1}{2}x = 6.30$$

$$\mathbf{2}\left(\frac{1}{2}x\right) = \mathbf{2}(6.30)$$ Multiply each side by 2.

$$x = 12.6$$