Solving Equations

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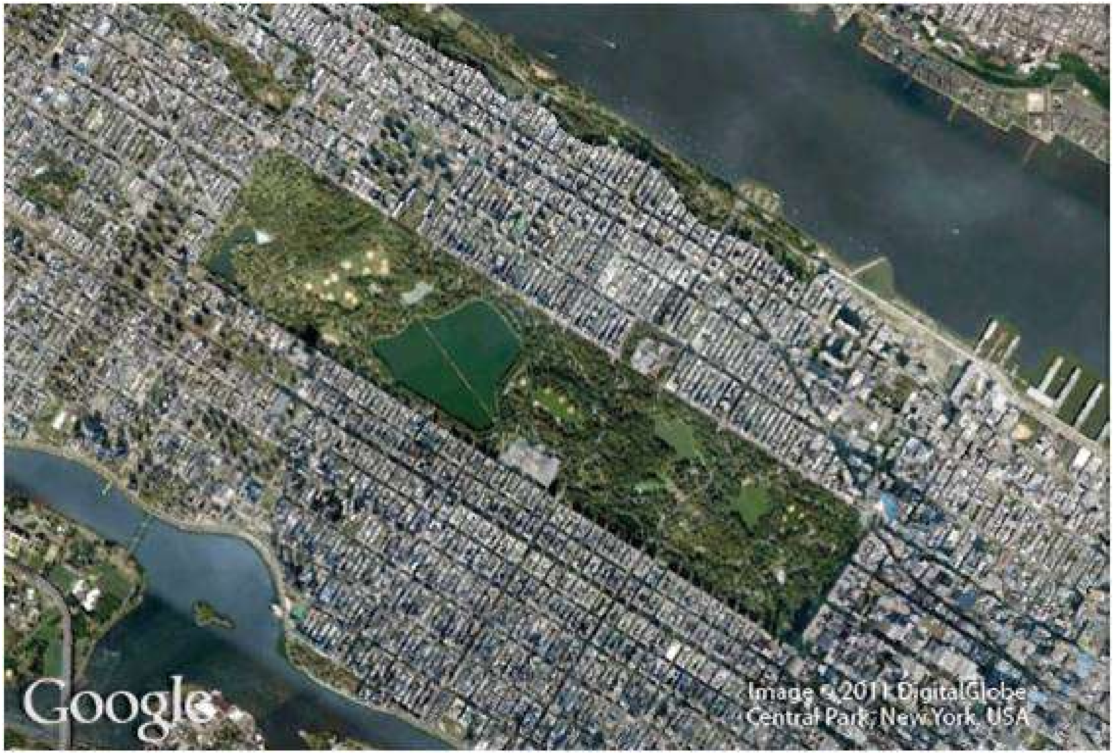

SECTION 4.3

The Multiplication Property of Equality

Objective

A Use the multiplication property of equality to solve equations.

The Multiplication Property of Equality

In this section, we will continue to solve equations in one variable.

We will again use the addition property of equality, but we will also use the multiplication property of equality to solve the equations in this section.

We will state the multiplication property of equality and then see how it is used by looking at some examples.

A Multiplication Property of Equality

Multiplication Property of Equality

Here is the formal explanation of the multiplication property of equality:

Multiplication Property of Equality

Let A, B, and C represent algebraic expressions, with C not equal to 0.

$$\text{If} \quad A = B$$
$$\text{then} \quad AC = BC$$

In words: Multiplying both sides of an equation by the same nonzero quantity will never change the solution to the equation.

Now, because division is defined as multiplication by the reciprocal, we are also free to divide both sides of an equation by the same nonzero quantity and always be sure we have not changed the solution to the equation.

Example 1

Solve for *x*: $\frac{1}{2}x = 3$

Solution:

We want to isolate *x* (that is, 1*x*) on one side of the equation.

We have $\frac{1}{2}x$ on the left side.

If we multiply both sides by 2, we will have 1*x* on the left side.

Example 1 – *Solution*

cont'd

Here is how it looks:

$$\frac{1}{2}x = 3$$

$$\mathbf{2}\left(\frac{1}{2}x\right) = \mathbf{2}(3) \qquad \text{Multiply both sides by 2.}$$

$$x = 6$$

To see why $2\left(\frac{1}{2}x\right)$ is equivalent to *x*, we use the associative property.

$$2\left(\frac{1}{2}x\right) = \left(2 \cdot \frac{1}{2}\right)x \qquad \text{Associative property}$$

Example 1 – *Solution*

cont'd

$$= 1 \cdot x \qquad 2 \cdot \frac{1}{2} = 1$$

$$= x \qquad 1 \cdot x = x$$

Although we will not show this step when solving problems, it is implied.

Multiplication Property of Equality

Note: The reciprocal of a negative number is also a negative number.

Remember, reciprocals are two numbers that have a product of 1.

Since 1 is a positive number, any two numbers we multiply to get 1 must both have the same sign.

Multiplication Property of Equality

Here are some negative numbers and their reciprocals:

The reciprocal of -2 is $-\frac{1}{2}$.

The reciprocal of -7 is $-\frac{1}{7}$.

The reciprocal of $-\frac{1}{3}$ is -3.

The reciprocal of $-\frac{3}{4}$ is $-\frac{4}{3}$.

The reciprocal of $-\frac{9}{5}$ is $-\frac{5}{9}$.

More about Sequences

More about Sequences

Now that we are familiar with finding the value of an algebraic expression, lets extend our work with sequences.

As the next example indicates, when we substitute the counting numbers, in order, into algebraic expressions, we form some of the sequences of numbers that you may have studied previously.

We know that the sequence of counting numbers (also called the sequence of positive integers) is

Counting numbers = 1, 2, 3, . . .

Example 9

Fill in the tables below to find the sequences formed by substituting the first four counting numbers into the expressions $2n$ and n^2.

a.

n	1	2	3	4
$2n$				

b.

n	1	2	3	4
n^2				

Solution:

We substitute the numbers 1, 2, 3, and 4 into the given expressions.

a. When $n = 1$, $2n = 2 \cdot 1 = 2$.

When $n = 2$, $2n = 2 \cdot 2 = 4$.

Example 9 – *Solution*

cont'd

When $n = 3$, $2n = 2 \cdot 3 = 6$.

When $n = 4$, $2n = 2 \cdot 4 = 8$.

As you can see, the expression $2n$ produces the sequence of even numbers when n is replaced by the counting numbers.

Placing these results into our first table gives us

n	1	2	3	4
$2n$	2	4	6	8

Example 9 – *Solution*

cont'd

b. The expression n^2 produces the sequence of squares when n is replaced by 1, 2, 3, and 4.

In table form, we have

n	1	2	3	4
n^2	1	4	9	16