Solving Equations

4

SECTION 4.2

The Addition Property of Equality

Objectives

A Identify a solution to an equation.

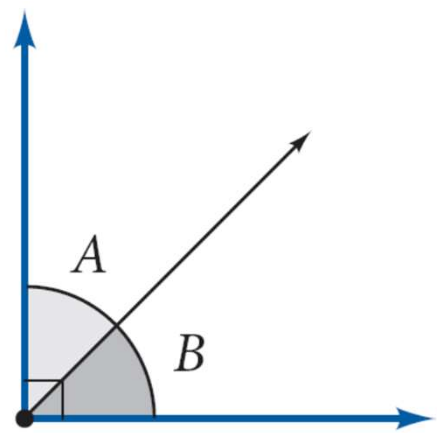
B Use the addition property of equality to solve linear equations.

The Addition Property of Equality

We have defined complementary angles as two angles whose sum is 90°.

If *A* and *B* are complementary angles, then

$$A + B = 90°$$



Complementary angles

The Addition Property of Equality

If we know that $A = 30°$, then we can substitute 30° for A in the formula above to obtain the equation

$$30° + B = 90°$$

In this section, we will learn how to solve equations like this one that involve addition and subtraction with one variable.

A Solutions to Equations

Solutions to Equations

When one expression is set equal to another, the result is called an equation. Here is a formal definition:

Definition

Two expressions of the same value separated by an equals sign is an **equation**.

We can solve an equation by finding the value for a variable that makes the equation a true statement. This value is called a *solution*.

Solutions to Equations

Definition

A **solution** for an equation is a number that when used in place of the variable makes the equation a true statement.

Example 1

Is $a = –2$ the solution to the equation $7a + 4 = 3a – 2$?

Solution:

To see if it is, we replace x with 5 in the equation and find out if the result is a true statement:

When $\rightarrow$ $a = –2$

the equation $\rightarrow$ $7a + 4 = 3a – 2$

becomes $\rightarrow$ $7(–2) + 4 = 3(–2) – 2$

Example 1 – *Solution*

cont'd

$$-14 + 4 = -6 - 2$$

$$-10 = -8$$ A false statement

Because the result is a false statement, we can conclude that $a = -2$ is *not* the solution to the equation $7a + 4 = 3a - 2$.

B Addition Property of Equality

Addition Property of Equality

We want to develop a process for solving equations with one variable. The most important property needed for solving the equations in this section is called the *addition property of equality*.

The formal definition looks like this:

Addition Property of Equality

Let A, B, and C represent algebraic expressions.

$$\begin{array}{rc} \text{If} & A = B \\ \text{then} & A + C = B + C \end{array}$$

In words: Adding the same quantity to both sides of an equation never changes the solution to the equation.

Addition Property of Equality

This property is extremely useful in solving equations.

Our goal in solving equations is to isolate the variable on one side of the equation.

We want to end up with an equation of the form

$$x = \text{a number}$$

To do so we use the addition property of equality.

Remember to follow this basic rule of algebra: *Whatever is done to one side of an equation must be done to the other side in order to preserve the equality*.

Example 2

Solve for x: $x + 4 = -2$

Solution:

We want to isolate x on one side of the equation.

If we add –4 to both sides, the left side will be $x + 4 + (-4)$, which is $x + 0$ or just x.

$$x + 4 = -2$$

$$x + 4 + \mathbf{(-4)} = -2 + \mathbf{(-4)} \qquad \text{Add } -4 \text{ to both sides.}$$

$$x + 0 = -6 \qquad \text{Add.}$$

$$x = -6 \qquad x + 0 = x$$

The solution is –6.

Example 2 – *Solution*

cont'd

We can check it if we want to, by replacing x with –6 in the original equation:

When $\rightarrow$	$x = -6$
the equation $\rightarrow$	$x + 4 = -2$
becomes $\rightarrow$	$-6 + 4 = -2$
	$-2 = -2$ A true statement

A Note on Subtraction

A Note on Subtraction

Although the addition property of equality is stated for addition only, we can subtract the same number from both sides of an equation as well.

Because subtraction is defined as addition of the opposite, subtracting the same quantity from both sides of an equation will not change the solution.

A Note on Subtraction

If we were to solve the equation $x + 4 = –2$ using subtraction instead of addition, the steps would look like this:

$x + 4 = –2$ Original equation

$x + 4 – \mathbf{4} = –2 – \mathbf{4}$ Subtract 4 from each side.

$x = –6$