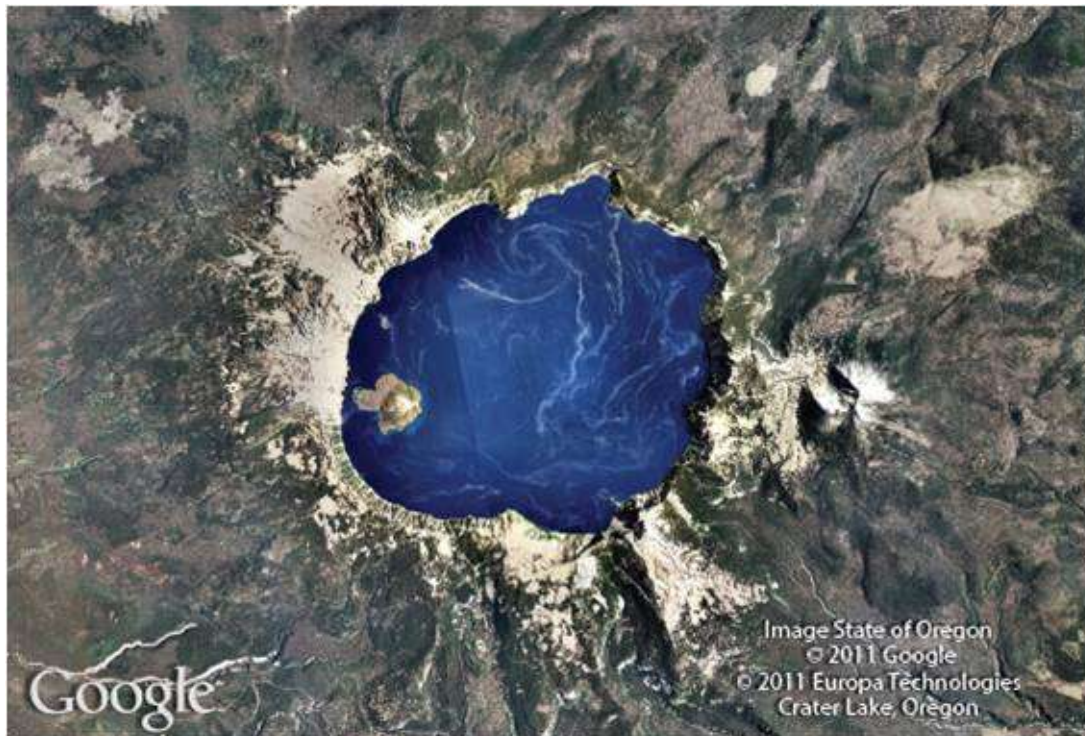


Fractions and Mixed Numbers

3



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SECTION 3.6

Mixed-Number Notation

Objectives

- A** Change mixed numbers to improper fractions.
- B** Change improper fractions to mixed numbers.

Mixed-Number Notation

The number $73\frac{5}{8}$ is called a *mixed number*.

It is simply the sum of the whole number, 73, and the proper fraction, $\frac{5}{8}$ written without a + sign.

With mixed-number notation, we leave out the addition sign.



Notation

Notation

Here are some further examples of mixed number notation:

$$2\frac{1}{8} = 2 + \frac{1}{8}, \quad 6\frac{5}{9} = 6 + \frac{5}{9}, \quad 11\frac{2}{3} = 11 + \frac{2}{3}$$

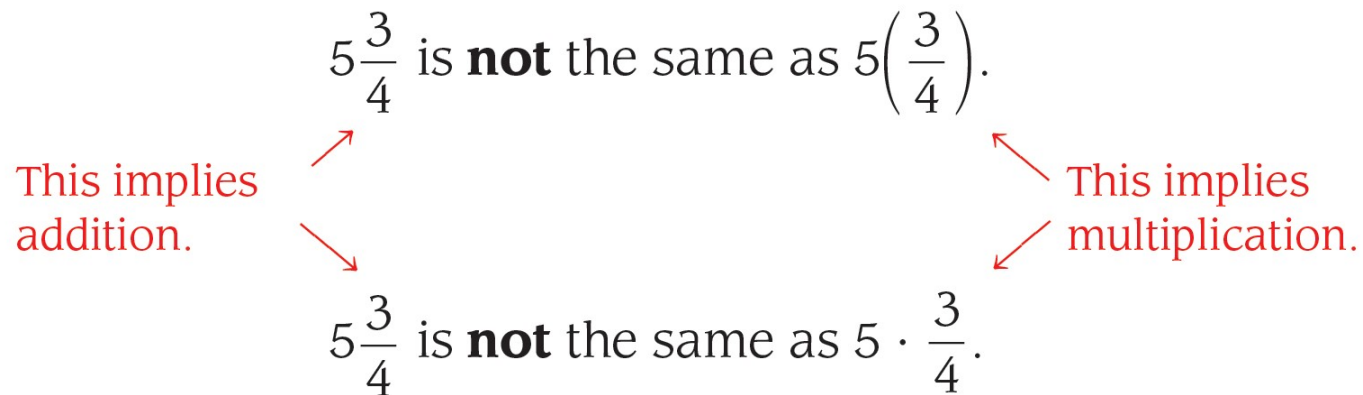
The notation used in writing mixed numbers (writing the whole number and the proper fraction next to each other) must always be interpreted as addition.

Notation

It is a mistake to read $5\frac{3}{4}$ as meaning 5 times $\frac{3}{4}$.

If we want to indicate multiplication, we must use parentheses or a multiplication symbol.

That is,





A Changing Mixed Numbers to Improper Fractions

Changing Mixed Numbers to Improper Fractions

To change a mixed number to an improper fraction, we write the mixed number with the + sign showing and then add the two numbers, as we did earlier.

Example 1

Change $2\frac{3}{4}$ to an improper fraction.

Solution:

$$2\frac{3}{4} = 2 + \frac{3}{4}$$

Write the mixed number as a sum.

$$= \frac{2}{1} + \frac{3}{4}$$

Show that the denominator of 2 is 1.

$$= \frac{4 \cdot 2}{4 \cdot 1} + \frac{3}{4}$$

Multiply the numerator and the denominator of $\frac{2}{1}$ by 4 so both fractions will have the same denominator.

$$= \frac{8}{4} + \frac{3}{4}$$

Example 1 – Solution

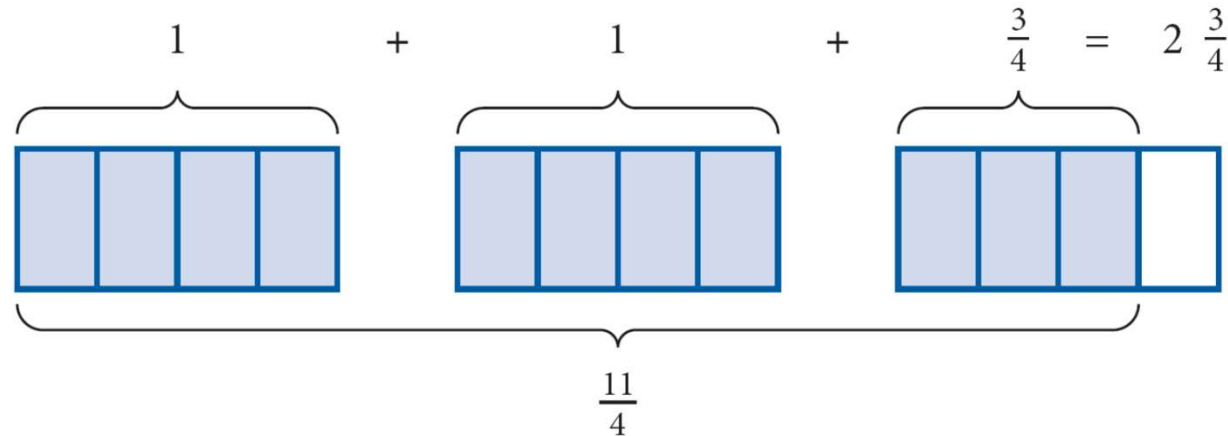
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$$= \frac{11}{4}$$

Add the numerators; keep the common denominator.

The mixed number $2\frac{3}{4}$ is equal to the improper fraction $\frac{11}{4}$.

The diagram that follows further illustrates the equivalence of $2\frac{3}{4}$ and $\frac{11}{4}$.



Changing Mixed Numbers to Improper Fractions

If we look closely at Example 1, we can see a shortcut that will let us change a mixed number to an improper fraction without so many steps.

Strategy Changing a Mixed Number to an Improper Fraction (Shortcut)

Step 1 Multiply the whole number part of the mixed number by the denominator.

Step 2 Add your answer to the numerator of the fraction.

Step 3 Put your new number over the original denominator.

Example 3

Use the shortcut to change $5\frac{3}{4}$ to an improper fraction.

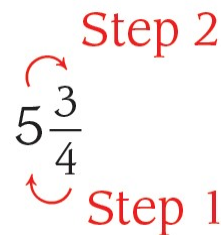
Solution:

1. First, we multiply 4×5 to get 20.
2. Next, we add 20 to 3 to get 23.
3. The improper fraction equal to $5\frac{3}{4}$ is $\frac{23}{4}$.

Here is a diagram showing what we have done:

Step 1 Multiply $4 \times 5 = 20$.

Step 2 Add $20 + 3 = 23$.



Example 3 – *Solution*

cont'd

Mathematically, our shortcut is written like this:

$$5\frac{3}{4} = \frac{(4 \cdot 5) + 3}{4}$$

$$= \frac{20 + 3}{4}$$

$$= \frac{23}{4}$$

The result will always have the same denominator as the original mixed number.

Changing Mixed Numbers to Improper Fractions

The shortcut shown in Example 3 works because the whole-number part of a mixed number can always be written with a denominator of 1.

Therefore, the LCD for a whole number and fraction will always be the denominator of the fraction.

That is why we multiply the whole number by the denominator of the fraction.

$$5\frac{3}{4} = 5 + \frac{3}{4} = \frac{5}{1} + \frac{3}{4} = \frac{4 \cdot 5}{4 \cdot 1} + \frac{3}{4} = \frac{4 \cdot 5 + 3}{4} = \frac{23}{4}$$



B Changing Improper Fractions to Mixed Numbers

Changing Improper Fractions to Mixed Numbers

To change an improper fraction to a mixed number, we divide the numerator by the denominator.

The result is used to write the mixed number.

Example 5

Change $\frac{11}{4}$ to a mixed number.

Solution:

Dividing 11 by 4 gives us

$$\begin{array}{r} 2 \\ 4 \overline{)11} \\ \underline{8} \\ 3 \end{array}$$

We see that 4 goes into 11 two times with 3 for a remainder.

Example 5 – *Solution*

cont'd

We write this as

$$\begin{aligned}\frac{11}{4} &= \mathbf{2} + \frac{3}{4} \\ &= 2\frac{3}{4}\end{aligned}$$

The improper fraction $\frac{11}{4}$ is equivalent to the mixed number $2\frac{3}{4}$.