Fractions and Mixed Numbers

3

SECTION 3.5

Addition and Subtraction with Fractions

Objectives

A Add and subtract fractions with the same denominator.

B Add and subtract fractions with different denominators.

A Combining Fractions with a Common Denominator

Combining Fractions with a Common Denominator

Adding and subtracting fractions is actually just another application of the distributive property.

The distributive property looks like this:

$$a(b + c) = a(b) + a(c)$$

where a, b, and c may be whole numbers or fractions.

We will want to apply this property to expressions like

$$\frac{2}{7} + \frac{3}{7}$$

Combining Fractions with a Common Denominator

But before we do, we must make one additional observation about fractions.

The fraction $\frac{2}{7}$ can be written as $2 \cdot \frac{1}{7}$, because

$$2 \cdot \frac{1}{7} = \frac{2}{1} \cdot \frac{1}{7} = \frac{2}{7}$$

Likewise, the fraction $\frac{3}{7}$ can be written as $3 \cdot \frac{1}{7}$, because

$$3 \cdot \frac{1}{7} = \frac{3}{1} \cdot \frac{1}{7} = \frac{3}{7}$$

Combining Fractions with a Common Denominator

In general, we can say that the fraction $\frac{a}{b}$ can always be written as $a \cdot \frac{1}{b}$, because

$$a \cdot \frac{1}{b} = \frac{a}{1} \cdot \frac{1}{b} = \frac{a}{b}$$

To add the fractions $\frac{2}{7}$ and $\frac{3}{7}$, we simply rewrite each of them as we have done above and apply the distributive property.

Combining Fractions with a Common Denominator

Here is how it works:

$$\frac{2}{7} + \frac{3}{7} = 2 \cdot \frac{1}{7} + 3 \cdot \frac{1}{7}$$ Rewrite each fraction.

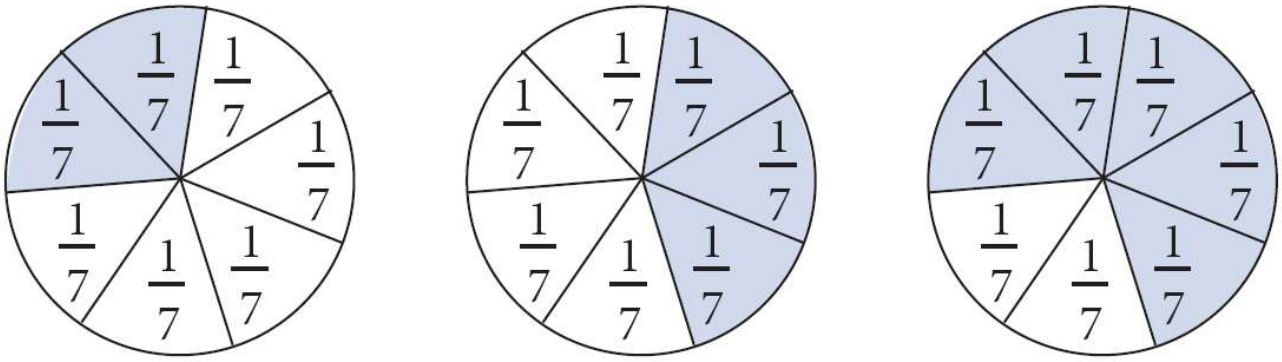
$$= (2 + 3) \cdot \frac{1}{7}$$ Apply the distributive property.

$$= 5 \cdot \frac{1}{7}$$ Add 2 and 3 to get 5.

$$= \frac{5}{7}$$ Rewrite $5 \cdot \frac{1}{7}$ as $\frac{5}{7}$.

Combining Fractions with a Common Denominator

We can visualize the process shown above by using circles that are divided into 7 equal parts:



$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$$

Combining Fractions with a Common Denominator

The fraction $\frac{5}{7}$ is the sum of $\frac{2}{7}$ and $\frac{3}{7}$.

The steps and diagrams above show why we add numerators *but do not add denominators*.

Using this example as justification, we can write a rule for adding two fractions that have the same denominator.

Rule **Addition with Common Denominator**

To add two fractions that have the same denominator, we add their numerators to get the numerator of the answer. The denominator in the answer is the same denominator as in the original fractions.

Combining Fractions with a Common Denominator

What we have here is the sum of the numerators placed over the *common denominator*.

In symbols we have the following:

Property **Addition and Subtraction of Fractions**

If a, b, and c are numbers, and c is not equal to 0, then

$$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$$

This rule holds for subtraction as well. That is,

$$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$$

Example 1

Add: $\frac{3}{8} + \frac{1}{8}$.

Solution:

$$\frac{3}{8} + \frac{1}{8} = \frac{3+1}{8}$$ Add numerators; keep the same denominator.

$$= \frac{4}{8}$$ The sum of 3 and 1 is 4.

$$= \frac{1}{2}$$ Reduce to lowest terms.

Example 2

Subtract: $\frac{a+5}{8} - \frac{3}{8}$

Solution:

$$\frac{a+5}{8} - \frac{3}{8} = \frac{a+5-3}{8}$$

Combine numerators; keep the same denominator.

$$= \frac{a+2}{8}$$

The difference of 5 and 3 is 2.

B The Least Common Denominator or LCD

The Least Common Denominator or LCD

We will now turn our attention to the process of adding fractions that have different denominators.

Look back at the table that began this section. How would we determine the difference in rainfall between Death Valley, California, and Wodi Halfa, Sudan? To begin answering this question, we need the following definition:

Definition

The **least common denominator** (LCD) for a set of denominators is the smallest number that is exactly divisible by each denominator. (Note that, in some books, the least common denominator is also called the *least common multiple*.)

The Least Common Denominator or LCD

In other words, all the denominators of the fractions involved in a problem must divide into the least common denominator exactly.

That is, they divide it without leaving a remainder.

Example 5

Find the LCD for the fractions $\frac{5}{12}$ and $\frac{7}{18}$.

Solution:

The least common denominator for the denominators 12 and 18 must be the smallest number divisible by both 12 and 18.

We can factor 12 and 18 completely and then build the LCD from these factors.

Factoring 12 and 18 completely gives us

$12 = 2 \cdot 2 \cdot 3$ $\qquad$ $18 = 2 \cdot 3 \cdot 3$

Example 5 – *Solution*

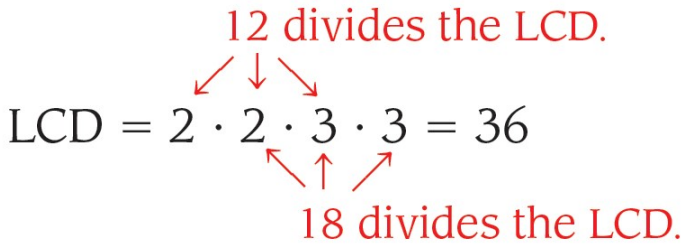
cont'd

Now, if 12 is going to divide the LCD exactly, then the LCD must have factors of $2 \cdot 2 \cdot 3$.

If 18 is to divide it exactly, it must have factors of $2 \cdot 3 \cdot 3$.

We don't need to repeat the factors that 12 and 18 have in common.

$$\left.\begin{aligned} 12 &= 2 \cdot 2 \cdot 3 \\ 18 &= 2 \cdot 3 \cdot 3 \end{aligned}\right\}$$



Example 5 – *Solution*

cont'd

The LCD for 12 and 18 is 36.

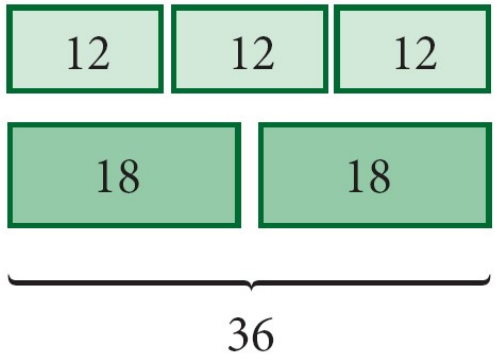
It is the smallest number that is divisible by both 12 and 18; 12 divides it exactly three times, and 18 divides it exactly two times.

The Least Common Denominator or LCD

We can visualize the results in Example 5 with the diagram below.

It shows that 36 is the smallest number that both 12 and 18 divide evenly.

As you can see, 12 divides 36 exactly 3 times, and 18 divides 36 exactly 2 times.



The Least Common Denominator or LCD

We now list the steps involved in adding and subtracting fractions with different denominators.

Strategy **Adding or Subtracting Any Two Fractions**

Step 1 Factor each denominator completely, and use the factors to build the LCD. (Remember, the LCD is the smallest number divisible by each of the denominators in the problem.)

Step 2 Rewrite each fraction as an equivalent fraction that has the LCD for its denominator. This is done by multiplying both the numerator and the denominator of the fraction in question by the appropriate whole number.

Step 3 Add or subtract the numerators of the fractions produced in Step 2. This is the numerator of the sum or difference. The denominator of the sum or difference is the LCD.

Step 4 Reduce the fraction produced in Step 3 to lowest terms if it is not already in lowest terms.

The Least Common Denominator or LCD

The idea behind adding or subtracting fractions is really very simple.

We can only add or subtract fractions that have the same denominators.

If the fractions we are trying to add or subtract do not have the same denominators, we rewrite each of them as an equivalent fraction with the LCD for a denominator.