Fractions and Mixed Numbers

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SECTION 3.4

Division with Fractions

Objectives

A Divide fractions.

B Simplify order of operations problems involving division of fractions.

C Solve application problems involving division of fractions.

Division with Fractions

A few years ago our 4-H club was making blankets to keep their lambs clean at the county fair.

Each blanket required $\frac{3}{4}$ yard of material. We had 9 yards of material left over from the year before.

To see how many blankets we could make, we divided 9 by $\frac{3}{4}$.

The result was 12, meaning that they could make 12 lamb blankets out of the 9 remaining yards.

Division with Fractions

Before we define division with fractions, we must first introduce the idea of *reciprocals*.

Look at the following multiplication problems:

$$\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12} = 1 \qquad \frac{7}{8} \cdot \frac{8}{7} = \frac{56}{56} = 1$$

In each case the product is 1.

Whenever the product of two numbers is 1, we say the two numbers are reciprocals.

Division with Fractions

Definition

Two numbers whose product is 1 are said to be **reciprocals.** In symbols, the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$, because

$$\frac{a}{b} \cdot \frac{b}{a} = \frac{a \cdot b}{b \cdot a} = \frac{a \cdot b}{a \cdot b} = 1 \qquad (a \neq 0, b \neq 0)$$

Every number has a reciprocal except 0.

The reason that 0 does not have a reciprocal is because the product of *any* number with 0 is 0. It can never be 1.

Reciprocals of whole numbers are fractions with 1 as the numerator.

Division with Fractions

For example, the reciprocal of 5 is $\frac{1}{5}$, because

$$5 \cdot \frac{1}{5} = \frac{5}{1} \cdot \frac{1}{5} = \frac{5}{5} = 1$$

Table 1 lists some numbers and their reciprocals.

TABLE 1

Number	Reciprocal	Reason
$\frac{3}{4}$	$\frac{4}{3}$	Because $\frac{3}{4} \cdot \frac{4}{3} = \frac{12}{12} = 1$
$\frac{9}{5}$	$\frac{5}{9}$	Because $\frac{9}{5} \cdot \frac{5}{9} = \frac{45}{45} = 1$
$\frac{1}{3}$	3	Because $\frac{1}{3} \cdot 3 = \frac{1}{3} \cdot \frac{3}{1} = \frac{3}{3} = 1$
7	$\frac{1}{7}$	Because $7 \cdot \frac{1}{7} = \frac{7}{1} \cdot \frac{1}{7} = \frac{7}{7} = 1$

A Dividing Fractions

Dividing Fractions

Division with fractions is accomplished by using reciprocals.

More specifically, we can define division by a fraction to be the same as multiplication by its reciprocal.

Here is the precise rule:

Rule **Dividing by a Fraction**

When dividing by a fraction, if a, b, c, and d are numbers and b, c, and d are not equal to 0, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

This definition states that dividing by the fraction $\frac{c}{d}$ is exactly the same as multiplying by its reciprocal $\frac{d}{c}$.

Example 1

Divide: $\frac{1}{2} \div \frac{1}{4}$

Solution:

The divisor is $\frac{1}{4}$, and its reciprocal is $\frac{4}{1}$.

Applying the definition of division for fractions, we have

$$\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \cdot \frac{4}{1}$$

$$= \frac{1 \cdot 4}{2 \cdot 1}$$

Example 1 – *Solution*

cont'd

$$= \frac{1 \cdot \cancel{2} \cdot 2}{\cancel{2} \cdot 1}$$

$$= \frac{2}{1}$$

$$= 2$$

The quotient of $\frac{1}{2}$ and $\frac{1}{4}$ is 2.

Or, $\frac{1}{4}$ "goes into" $\frac{1}{2}$ two times.

Example 1 – *Solution*

cont'd

Logically, our definition for division of fractions seems to be giving us answers that are consistent with what we know about fractions from previous experience.

Because 2 times $\frac{1}{4}$ is $\frac{2}{4}$ or $\frac{1}{2}$, it seems logical that $\frac{1}{2}$ divided by $\frac{1}{4}$ should be 2.

B Fractions and the Order of Operations

Example 9

The quotient of $\frac{8}{3}$ and $\frac{1}{6}$ is increased by 5. What number results?

Solution:

Translating to symbols, we have

$$\frac{8}{3} \div \frac{1}{6} + 5 = \frac{8}{3} \cdot \frac{6}{1} + 5$$

$$= 16 + 5$$

$$= 21$$