Fractions and Mixed Numbers

3

SECTION 3.2

Prime Numbers, Factors, and Reducing to Lowest Terms

Objectives

A Identify numbers as prime or composite.

B Factor a number into the product of prime factors.

C Write a fraction in lowest terms.

A Prime Numbers

Prime Numbers

Definition

A **prime number** is any whole number greater than 1 that has exactly two divisors—the number 1 and itself. (Remember a number is a divisor of another number if it divides it without a remainder.)

Definition

Any whole number greater than 1 that is not a prime number is called a **composite number.** A composite number always has at least one divisor other than the number 1 and itself.

Example 1

Identify each of the numbers below as either a prime number or a composite number. For those that are composite, give two divisors other than the number itself or 1.

a. 43 **b.** 12

Solution:

a. 43 is a prime number, because the only numbers that divide it without a remainder are 43 and 1.

b. 12 is a composite number, because it can be written as $12 = 4 \cdot 3$, which means that 4 and 3 are divisors of 12. (These are not the only divisors of 12; other divisors are 1, 2, 6, and 12.)

B Factoring

Factoring

Every composite number can be written as the *product of prime factors*.

Let's look at the composite number 108. We know we can write 108 as $2 \cdot 54$.

The number 2 is a prime number, but 54 is not prime. Because 54 can be written as $2 \cdot 27$, we have

$$\begin{aligned} 108 &= 2 \cdot 54 \\ &= 2 \cdot 2 \cdot 27 \end{aligned}$$

Factoring

Now the number 27 can be written as $3 \cdot 9$ or $3 \cdot 3 \cdot 3$ (because $9 = 3 \cdot 3$), so

$$108 = 2 \cdot 54$$

$$108 = 2 \cdot 2 \cdot 27$$

$$108 = 2 \cdot 2 \cdot 3 \cdot 9$$

$$108 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 3$$

This last line is the number 108 written as the product of prime factors. We can use exponents to rewrite the last line:

$$108 = 2^2 \cdot 3^3$$

Example 2

Factor 60 into a product of prime factors.

Solution:

We begin by writing 60 as 6 · 10 and continue factoring until all factors are prime numbers.

$$60 = 6 \cdot 10$$

$$= 2 \cdot 3 \cdot 2 \cdot 5$$

$$= 2^2 \cdot 3 \cdot 5$$

Example 2 – *Solution*

cont'd

Notice that if we had started by writing 60 as 3 · 20, we would have achieved the same result.

$$
\begin{aligned}
60 &= 3 \cdot 20 \\
&= 3 \cdot 2 \cdot 10 \\
&= 3 \cdot 2 \cdot 2 \cdot 5 \\
&= 2^2 \cdot 3 \cdot 5
\end{aligned}
$$

C Reducing Fractions

Reducing Fractions

We can use the method of factoring numbers into prime factors to help reduce fractions to lowest terms. Here is the definition for *lowest terms:*

Definition

A fraction is said to be in **lowest terms** if the numerator and the denominator have no factors in common other than the number 1.

Example 3

The fractions $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5},$ and $\frac{4}{5}$ are all in lowest terms, because in each case the numerator and the denominator have no factors other than 1 in common.

That is, in each fraction, no number other than 1 divides both the numerator and the denominator exactly (without a remainder).

Reducing Fractions

Reducing a fraction to lowest terms is simply a matter of dividing the numerator and the denominator by all the factors they have in common.

Example 5

Reduce the fraction $\frac{12}{15}$ to lowest terms by first factoring the numerator and the denominator into prime factors and then dividing both the numerator and the denominator by the factor they have in common.

Solution:

The numerator and the denominator factor as follows:

$$12 = 2 \cdot 2 \cdot 3 \quad \text{and } 15 = 3 \cdot 5$$

The factor they have in common is 3.

Example 5 – *Solution*

cont'd

Property 2 tells us that we can divide both terms of a fraction by 3 to produce an equivalent fraction.

$$\frac{12}{15} = \frac{2 \cdot 2 \cdot 3}{3 \cdot 5}$$

Factor the numerator and the denominator completely.

$$= \frac{2 \cdot 2 \cdot 3 \div \mathbf{3}}{3 \cdot 5 \div \mathbf{3}}$$

Divide by 3.

$$= \frac{2 \cdot 2}{5} = \frac{4}{5}$$

The fraction $\frac{4}{5}$ is equivalent to $\frac{12}{15}$ and is in lowest terms, because the numerator and the denominator have no factors other than 1 in common.

Reducing Fractions

We can shorten the work involved in reducing fractions to lowest terms by using a slash to indicate division.

For example, we can write the above problem as

$$\frac{12}{15} = \frac{2 \cdot 2 \cdot \cancel{3}}{\cancel{3} \cdot 5} = \frac{4}{5}$$

So long as we understand that the slashes through the 3s indicate that we have divided both the numerator and the denominator by 3, we can use this notation.

Fractions with Variables

Fractions with Variables

Furthermore, we can apply what we know about reducing fractions to lowest terms to algebra. If a fraction contains variables (letters) in its numerator and/or denominator, we treat the variables in the same way we treat the other numbers in the numerator or denominator.

Example 9

Reduce $\frac{36xy}{120x}$ to lowest terms.

Solution:

We begin by factoring both terms. We then divide through by any factors common to both terms.

$$\frac{36xy}{120x} = \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{3} \cdot 3 \cdot \cancel{x} \cdot y}{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{3} \cdot 5 \cdot \cancel{x}}$$

$$= \frac{3y}{10}$$