Introduction to Algebra

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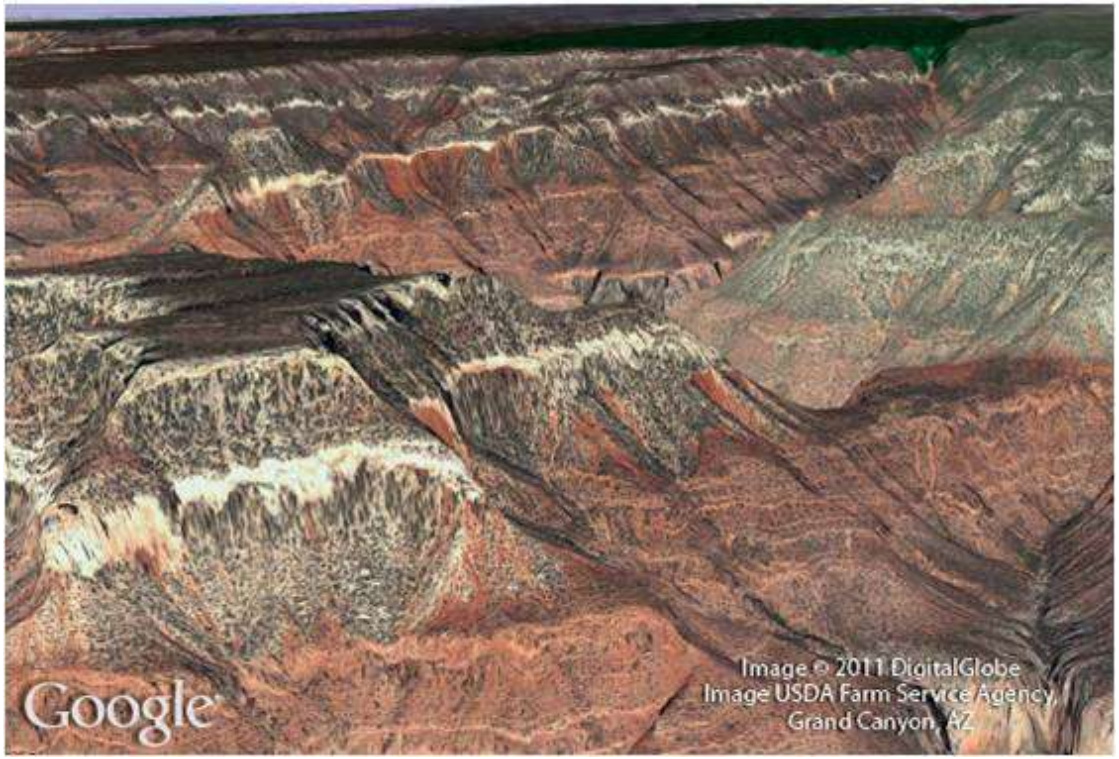


SECTION 2.6

Simplifying Algebraic Expressions

Objectives

A Simplify expressions by using the associative property.

B Apply the distributive property to expressions containing numbers and variables.

C Use the distributive property to combine similar terms.

D Use the formulas for area and perimeter of squares and rectangles.

Simplifying Algebraic Expressions

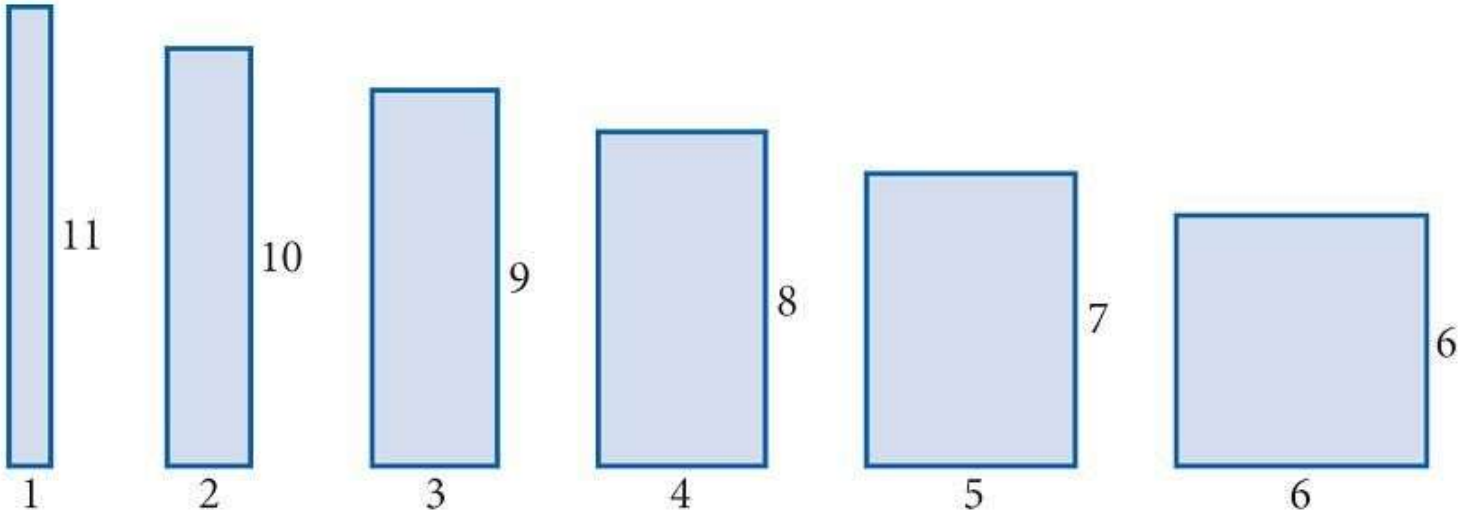
Suppose we have 24 yards of fencing that we are to use to build a rectangular dog run.

If we want the dog run to have the largest area possible then we want the rectangle, with perimeter 24 yards, that encloses the largest area.

Simplifying Algebraic Expressions

The diagram below shows six dog runs, each of which has a perimeter of 24 yards. Notice how the length decreases as the width increases.

Dog Runs with Perimeter = 24 yards



Simplifying Algebraic Expressions

In this section we want to simplify expressions containing variables—that is, algebraic expressions.

An algebraic expression is a combination of constants and variables joined by arithmetic operations such as addition, subtraction, multiplication and division.

A Using the Associative Property

Using the Associative Property

To begin let's review how we use the associative properties for addition and multiplication to simplify expressions.

Consider the expression $4(5x)$.

We can apply the associative property of multiplication to this expression to change the grouping so that the 4 and the 5 are grouped together, instead of the 5 and the x.

Using the Associative Property

Here's how it looks:

$$4(5x) = (4 \cdot 5)x \qquad \text{Associative property}$$

$$= 20x \qquad \text{Multiply: } 4 \cdot 5 = 20.$$

We have simplified the expression to $20x$, which in most cases in algebra will be easier to work with than the original expression.

Examples

Example 1

$7(3a) = (7 \cdot 3)a$ Associative property

$= 21a$ 7 times 3 is 21.

We can use the associative property of addition to simplify expressions also.

Example 4

$3 + (8 + x) = (3 + 8) + x$ Associative property

$= 11 + x$ The sum of 3 and 8 is 11.

B Using the Distributive Property

Using the Distributive Property

We have introduced the distributive property.

In symbols it looks like this:

$$a(b + c) = ab + ac$$

Because subtraction is defined as addition of the opposite, the distributive property holds for subtraction as well as addition.

That is,

$$a(b - c) = ab - ac$$

We say that multiplication distributes over addition and subtraction.

Example 6

$4(x + 5) = 4(x) + 4(5)$ Distributive property

$= 4x + 20$ Multiply.

Using the Distributive Property

In Example 1 we have simplified expression such as $4(5x)$ by using the associative property.

The following example uses a combination of the associative property and the distributive property.

Example 8

$4(5x + 3) = 4(5x) + 4(3)$ Distributive property

$= (4 \cdot 5)x + 4(3)$ Associative property

$= 20x + 12$ Multiply.

c Similar Terms

Similar Terms

We can also use the distributive property to simplify expressions like $4x + 3x$.

Because multiplication is a commutative operation, we can also rewrite the distributive property like this:

$$b \cdot a + c \cdot a = (b + c)a$$

Applying the distributive property in this form to the expression $4x + 3x$, we have

$$4x + 3x = (4 + 3)x \qquad \text{Distributive property}$$

$$= 7x \qquad \text{Add.}$$

Similar Terms

Expressions like $4x$ and $3x$ are called *similar terms* because the variable parts are the same.

To simplify an algebraic expression (an expression that involves both numbers and variables), we combine similar terms by applying the distributive property.

Similar Terms

Table 1 shows several pairs of similar terms and how they can be combined using the distributive property.

TABLE 1

Original Expression		Apply Distributive Property		Simplified Expression
$4x + 3x$	=	$(4 + 3)x$	=	$7x$
$7a + a$	=	$(7 + 1)a$	=	$8a$
$-5x + 7x$	=	$(-5 + 7)x$	=	$2x$
$8y - y$	=	$(8 - 1)y$	=	$7y$
$-4a - 2a$	=	$(-4 - 2)a$	=	$-6a$
$3x - 7x$	=	$(3 - 7)x$	=	$-4x$

As you can see from the table, the distributive property can be applied to any combination of positive and negative terms so long as they are similar terms.

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Algebraic Expressions Representing Area and Perimeter

Algebraic Expressions Representing Area and Perimeter

Below are a square with a side of length *s* and a rectangle with a length of *l* and a width of *w*. Table 2 gives the formulas for the area and perimeter of each.

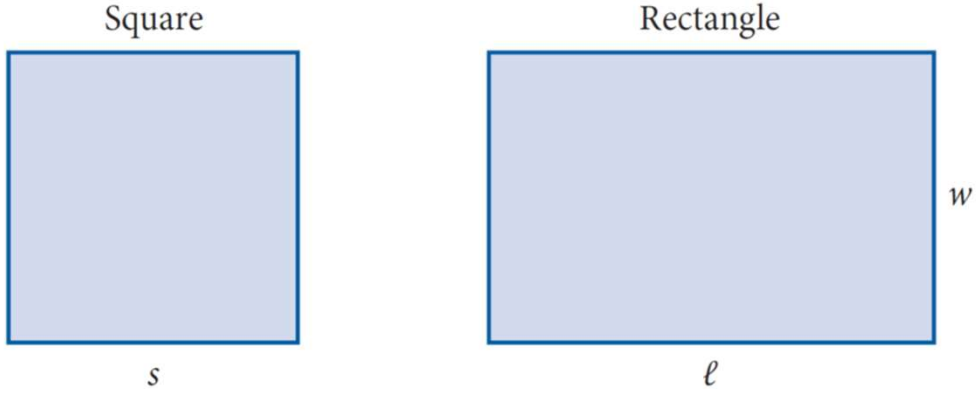

TABLE 2

	Square	Rectangle
Area A	s^2	ℓw
Perimeter P	$4s$	$2\ell + 2w$

Example 13

Find the area and perimeter of a square with a side 6 inches long.

Solution:

Substituting 6 for s in the formulas for area and perimeter of a square, we have

$$\text{Area} = A = s^2 = 6^2 = 36 \text{ square inches}$$

$$\text{Perimeter} = P = 4s = 4(6) = 24 \text{ inches}$$