Introduction to Algebra

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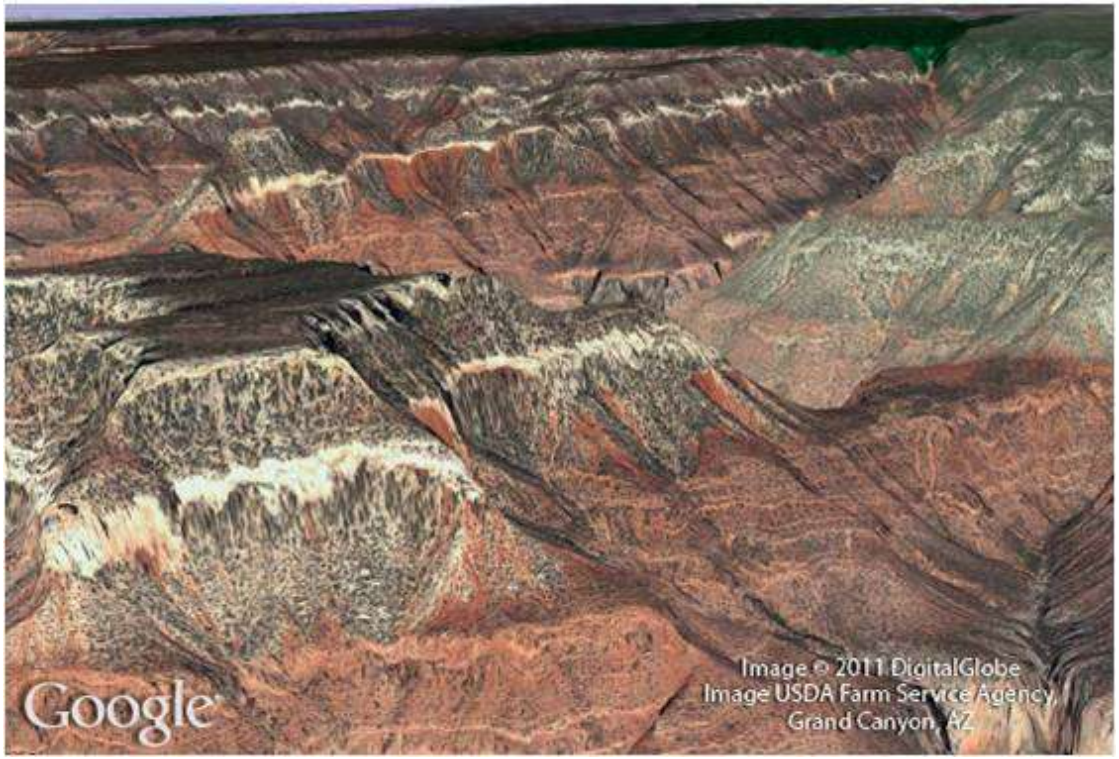

SECTION 2.5

Division with Negative Numbers

Objectives

A Divide positive and negative numbers.

B Apply the rule for order of operations to expressions that contain positive and negative numbers.

Division with Negative Numbers

Suppose four friends invest equal amounts of money in a moving truck to start a small business.

After 2 years the truck has dropped $10,000 in value.

If we represent this change with the number –$10,000, then the loss to each of the four partners can be found with division:

$$(–\$10{,}000) \div 4 = –\$2{,}500$$

Division with Negative Numbers

From this example it seems reasonable to assume that a negative number divided by a positive number will give a negative answer.

A Division with Negatives

Division with Negatives

To cover all the possible situations we can encounter with division of negative numbers, we use the relationship between multiplication and division. If we let n be the answer to the problem $12 \div (-2)$, then we know that

$$12 \div (-2) = n \text{ and } -2(n) = 12$$

From our work with multiplication, we know that n must be –6 in the multiplication problem above, because –6 is the only number we can multiply –2 by to get 12.

Because of the relationship between the two problems above, it must be true that 12 divided by –2 is –6.

Division with Negatives

The following pairs of problems show more quotients of positive and negative numbers.

In each case the multiplication problem on the right justifies the answer to the division problem on the left.

$$6 \div 3 = 2 \quad \text{because} \quad 3(2) = 6$$
$$6 \div (-3) = -2 \quad \text{because} \quad -3(-2) = 6$$
$$-6 \div 3 = -2 \quad \text{because} \quad 3(-2) = -6$$
$$-6 \div -3 = 2 \quad \text{because} \quad -3(2) = -6$$

The results given above can be used to write the rule for division with negative numbers.

Division with Negatives

Rule **Division of Any Two Numbers**

To divide any two numbers, we divide their absolute values.

1. The answer is *positive* if both the original numbers have the same sign. That is, the quotient of two numbers with the same signs is positive.
2. The answer is *negative* if the original two numbers have different signs. That is, the quotient of two numbers with different signs is negative.

Examples

Example 1

$-12 \div 4 = -3$ Unlike signs, negative answer

Example 2

$-12 \div (-4) = 3$ Like signs; positive answer

Example 3

$$\frac{-20}{-4} = 5$$ Like signs; positive answer

Division with Negatives

From the examples we have done so far, we can make the following generalization about quotients that contain negative signs:

If a and b are numbers and b is not equal to 0, then

$$-\frac{a}{b} = \frac{a}{-b} = \frac{-a}{b} \quad \text{and} \quad \frac{-a}{-b} = \frac{a}{b}$$

B Order of Operations

Example 6

Simplify: $\frac{6(-3)}{-2}$

Solution:

We begin by multiplying 6 and –3:

$$\frac{6(-3)}{-2} = \frac{-18}{-2}$$ Multiply: $6(-3) = -18$.

$$= 9$$ Like signs; positive answer