

Introduction to Algebra

2



SECTION 2.4

Multiplication with Negative Numbers

Objectives

- A** Multiply positive and negative numbers.
- B** Apply the rule for order of operations to expressions containing positive and negative numbers.

Multiplication with Negative Numbers

Suppose you buy three shares of a certain stock on Monday, and by Friday the price per share has dropped \$5. How much money have you lost?

The answer is \$15.

Because it is a loss, we can express it as $-\$15$.

Multiplication with Negative Numbers

The multiplication problem below can be used to describe the relationship among the numbers.

3 shares each loses \$5 for a total of -\$15.

$$3(-5) = -15$$

From this we conclude that it is reasonable to say that the product of a positive number and a negative number is a negative number.



A Multiplication with Negatives

Multiplication with Negatives

In order to generalize multiplication with negative numbers, recall that we first defined multiplication by whole numbers to be repeated addition. That is:

$$\begin{array}{c} 3 \cdot 5 = 5 + 5 + 5 \\ \uparrow \qquad \qquad \uparrow \\ \text{Multiplication} \quad \text{Repeated addition} \end{array}$$

This concept is very helpful when it comes to developing the rule for multiplication problems that involve negative numbers.

Example 1

Multiply: $3(-5)$

Solution:

Writing this product as repeated addition, we have

$$3(-5) = (-5) + (-5) + (-5)$$

$$= -10 + (-5)$$

$$= -15$$

The result, -15 , is obtained by adding the three negative 5s.

Example 2

Multiply: $-3(5)$

Solution:

In order to write this multiplication problem in terms of repeated addition, we will have to reverse the order of the two numbers.

This is easily done, because multiplication is a commutative operation.

$$\begin{aligned} -3(5) &= 5(-3) && \text{Commutative property} \\ &= (-3) + (-3) + (-3) + (-3) + (-3) && \text{Repeated addition} \\ &= -15 && \text{Add.} \end{aligned}$$

The product of -3 and 5 is -15 .

Example 3

Multiply: $-3(-5)$

Solution:

It is impossible to write this product in terms of repeated addition.

We will find the answer to $-3(-5)$ by solving a different problem.

Example 3 – *Solution*

cont'd

Look at the following problem:

$$\begin{aligned} -3[5 + (-5)] &= -3[0] \\ &= 0 \end{aligned}$$

The result is 0, because multiplying by 0 always produces 0.

Now we can work the same problem another way, and in the process find the answer to $-3(-5)$.

Example 3 – *Solution*

cont'd

Applying the distributive property to the same expression, we have

$$\begin{aligned} -3[5 + (-5)] &= -3(5) + (-3)(-5) && \text{Distributive property} \\ &= -15 + (?) && -3(5) = -15 \end{aligned}$$

The question mark must be 15, because we already know that the answer to the problem is 0, and 15 is the only number we can add to -15 to get 0.

So, our problem is solved:

$$-3(-5) = 15$$

Multiplication with Negatives

Table 1 gives a summary of what we have done so far in this section.

TABLE 1			
Original Numbers Have		For Example	The Answer Is
Same signs		$3(5) = 15$	Positive
Different signs		$-3(5) = -15$	Negative
Different signs		$3(-5) = -15$	Negative
Same signs		$-3(-5) = 15$	Positive

Multiplication with Negatives

From the examples we have done so far in this section and their summaries in Table 1, we write the following rule for multiplication of positive and negative numbers:

Rule Multiplication of Any Two Numbers

To multiply any two numbers, we multiply their absolute values.

1. The answer is *positive* if both the original numbers have the same sign. That is, the product of two numbers with the same sign is positive.
2. The answer is *negative* if the original two numbers have different signs. The product of two numbers with different signs is negative.



B Order of Operations

Order of Operations

Now let's apply the rule for order of operations to the following examples that involve positive and negative numbers.

Remember, the rule for order of operations specifies that we are to work inside the parentheses first, and then simplify numbers containing exponents.

After this, we multiply and divide, left to right. The last step is to add and subtract, left to right.

Example 12

Simplify: $-6[3 + (-5)]$

Solution:

We begin inside the brackets and work our way out:

$$\begin{aligned} -6[3 + (-5)] &= -6[-2] \\ &= 12 \end{aligned}$$