

Introduction to Algebra

2



Copyright © Cengage Learning. All rights reserved.

SECTION 2.2

Addition with Negative Numbers

Objectives

- A** Use the number line to add positive and negative numbers.
- B** Add positive and negative numbers using a rule.

Addition with Negative Numbers

Suppose you are in Las Vegas playing blackjack and you lose \$3 on the first hand and then you lose \$5 on the next hand.

If you represent winning with positive numbers and losing with negative numbers, how will you represent the results from your first two hands?

Since you lost \$3 and \$5 for a total of \$8, one way to represent the situation is with addition of negative numbers:

$$(-\$3) + (-\$5) = -\$8$$

Addition with Negative Numbers

From this example we see that the sum of two negative numbers is a negative number.

To generalize addition of positive and negative numbers, we can use the number line.



A Adding with a Number Line

Adding with a Number Line

We can think of each number on the number line as having two characteristics:

- (1) a *distance* from 0 (absolute value) and
- (2) a *direction* from 0 (positive or negative).

The distance from 0 is represented by the numerical part of the number (like the 5 in the number -5), and its direction is represented by the $+$ or $-$ sign in front of the number.

We can visualize addition of numbers on the number line by thinking in terms of distance and direction from 0.

Adding with a Number Line

Let's begin with a simple problem we know the answer to. We interpret the sum $3 + 5$ on the number line as follows:

1. The first number is 3, which tells us “start at the origin, and move 3 units in the positive direction.”
2. The $+$ sign is read “and then move.”
3. The 5 means “5 units in the positive direction.”

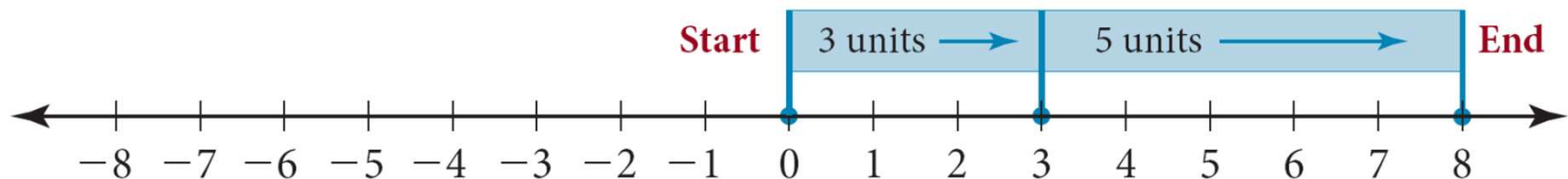


Figure 1

Figure 1 shows these steps.

Adding with a Number Line

To summarize, $3 + 5$ means to start at the origin (0), move 3 units in the *positive* direction, and then move 5 units in the *positive* direction.

We end up at 8, which is the sum we are looking for:

$$3 + 5 = 8.$$

Example 1

Add $3 + (-5)$ using the number line.

Solution:

We start at the origin, move 3 units in the positive direction, and then move 5 units in the negative direction, as shown in Figure 2.

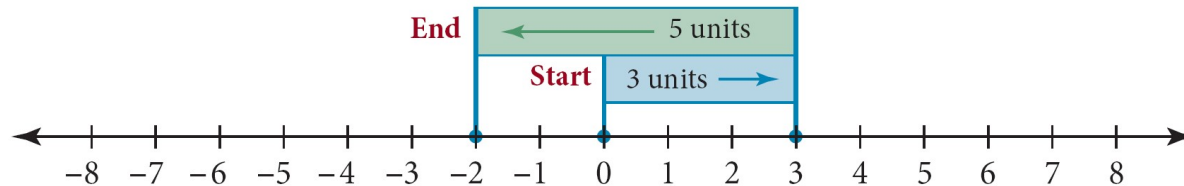


Figure 2

The last arrow ends at -2 , which must be the sum of 3 and -5 . That is:

$$3 + (-5) = -2$$

Adding with a Number Line

Adding numbers on the number line as we have done in Example 1 gives us a way of visualizing addition of positive and negative numbers.

We eventually want to be able to write a rule for addition of positive and negative numbers that doesn't involve the number line.

The number line is a way of justifying the rule we will eventually write.

Here is a summary of the results we have so far:

$$\begin{array}{ll} 3 + 5 = 8 & -3 + 5 = -2 \\ 3 + (-5) = -2 & -3 + (-5) = -8 \end{array}$$



B A Rule for Addition

A Rule for Addition

We can write the following rule for adding any two numbers:

Rule Addition of Any Two Numbers

1. To add two numbers with the *same* sign: Simply add their absolute values, and use the common sign. If both numbers are positive, the answer is positive. If both numbers are negative, the answer is negative.
2. To add two numbers with *different* signs: Subtract the smaller absolute value from the larger absolute value. The answer will have the sign of the number with the larger absolute value.

Example 8

Add all combinations of positive and negative 10 and 15.

Solution:

$$10 + 15 = 25$$

$$10 + (-15) = -5$$

$$-10 + 15 = 5$$

$$-10 + (-15) = -25$$

A Rule for Addition

Notice that when we add two numbers with the same sign, the answer also has that sign.

When the signs are not the same, the answer has the sign of the number with the larger absolute value.