Introduction to Algebra

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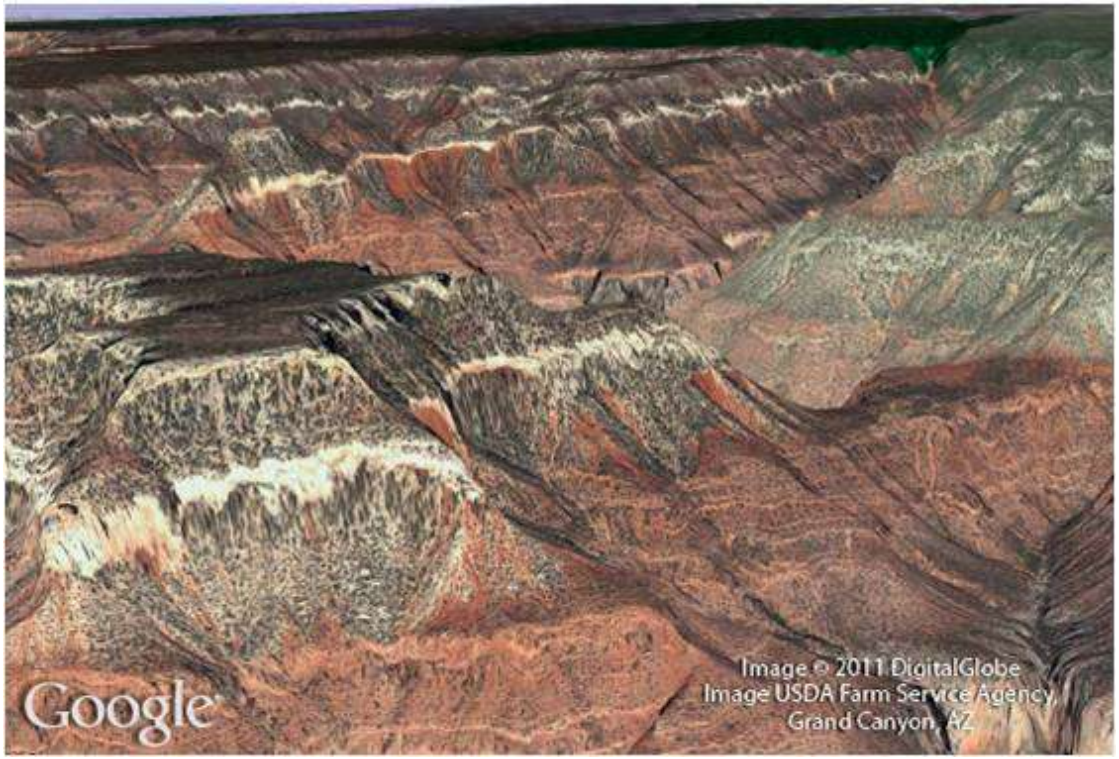


SECTION 2.1

Positive and Negative Numbers

Objectives

A Use the number line and inequality symbols to compare numbers.

B Find the absolute value of a number.

C Find the opposite of a number.

Positive and Negative Numbers

Before the late nineteenth century, time zones did not exist.

Each town would set its clocks according to the motions of the Sun.

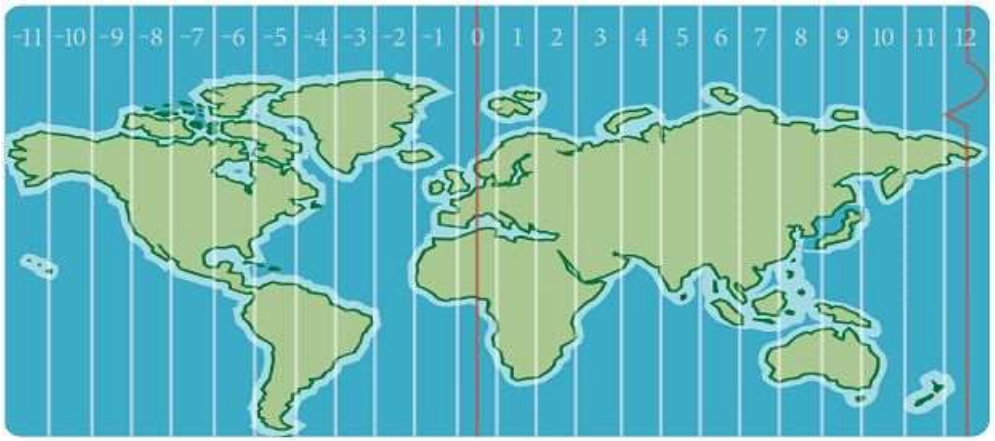
It was not until the late 1800s that a system of worldwide time zones was developed.

This system divides the earth into 24 time zones with Greenwich, England designated as the center of the time zones (GMT).

Positive and Negative Numbers

This location is assigned a value of zero.

Each of the World Time Zones is assigned a number ranging from –12 to +12 depending on its position east or west of Greenwich, England.



If New York is 5 time zones to the left of GMT, this would be noted as –5:00 GMT.

A Comparing Numbers

Comparing Numbers

To see the relationship between negative and positive numbers, we can extend the number line as shown in Figure 1.

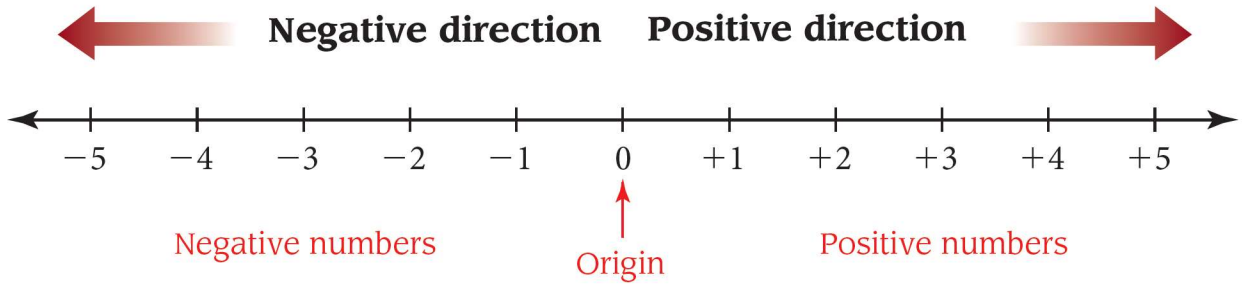


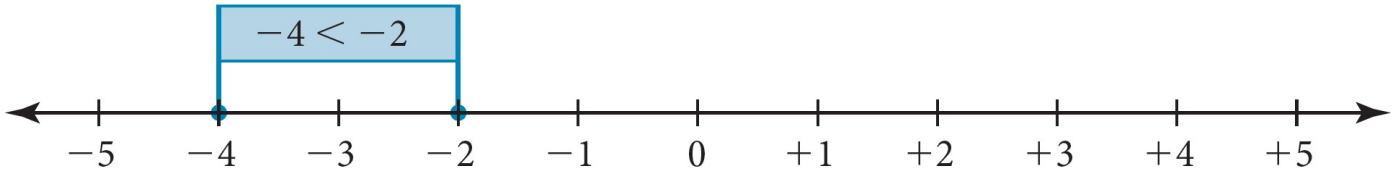
Figure 1

We first draw a straight line and label a convenient point with 0. This is called the *origin*, and it is usually in the middle of the line. We then label positive numbers to the right and negative numbers to the left.

Comparing Numbers

The numbers increase going from left to right. If we move to the right, we are moving in the positive direction. If we move to the left, we are moving in the negative direction.

Any number to the left of another number is considered to be smaller than the number to its right.



−4 is less than −2 because −4 is to the left of −2 on the number line

Figure 2

We see from the line that every negative number is less than every positive number.

Comparing Numbers

In algebra we can use inequality symbols when comparing numbers.

Notation **Inequalities**

For any whole numbers a and b

1. $a < b$ is read "a is less than b" and is true when a is to the **left** of b on a number line
2. $a > b$ is read "a is greater than b" and is true when a is to the **right** of b on a number line

As you can see, the inequality symbols always point to the smaller of the two numbers being compared.

Example 1

$3 < 5$ is read “3 is less than 5.”

Note that it would also be correct to write $5 > 3$.

Both statements, “3 is less than 5” and “5 is greater than 3,” have the same meaning.

The inequality symbols always point to the smaller number.

B Absolute Value

Absolute Value

It is sometimes convenient to talk about only the numerical part of a number and disregard the sign (+ or –) in front of it.

The following definition gives us a way of doing this.

Definition

The **absolute value** of a number is its distance from 0 on the number line. We denote the absolute value of a number with vertical lines. For example, the absolute value of -3 is written $|-3|$.

The absolute value of a number is never negative because it is a distance, and a distance is always measured in positive units (unless it happens to be 0).

Examples

Example 5

$|5| = 5$

The number 5 is 5 units from 0.

Example 6

$|-3| = 3$

The number –3 is 3 units from 0.

c Opposites

Opposites

Definition

Two numbers that are the same distance from 0 but in opposite directions from 0 are called **opposites.** The notation for the opposite of a is $-a$.

Example 8

Give the opposite of each of the following numbers:

5, 7, 1, –5, –8

Solution:

The opposite of 5 is –5.

The opposite of 7 is –7.

The opposite of 1 is –1.

The opposite of –5 is –(–5), or 5.

The opposite of –8 is –(–8), or 8.

Opposites

We see from Example 8 that the opposite of every positive number is a negative number, and likewise, the opposite of every negative number is a positive number.

Example 8 illustrate the following property:

Property **Opposite of a Negative**

If a represents any positive number, then it is *always* true that

$$-(-a)=a$$

In other words, this property states that the opposite of a negative number is a positive number.

Opposites

It should be evident now that the symbols + and – can be used to indicate several different ideas in mathematics.

In the past we have used them to indicate addition and subtraction.

They can also be used to indicate the direction a number is from 0 on the number line.

Opposites

For instance, the number +3 (read “positive 3”) is the number that is 3 units from zero in the positive direction.

On the other hand, the number –3 (read “negative 3”) is the number that is 3 units from 0 in the negative direction.

The symbol – can also be used to indicate the opposite of a number, as in –(–2) = 2.

The interpretation of the symbols + and – depends on the situation in which they are used.

Opposites

For example:

3 + 5	The + sign indicates addition.
7 – 2	The – sign indicates subtraction.
–7	The – sign is read “negative” 7.
–(–5)	The first – sign is read “the opposite of.” The second – sign is read “negative” 5.

Opposites

We should mention here that the set of whole numbers along with their opposites forms the set of *integers*.

That is:

Integers = {. . . , –3, –2, –1, 0, 1, 2, 3, . . .}