Whole Numbers

1

SECTION 1.8

Area and Volume

Objectives

A Find the area of a polygon.

B Find the volume of an object.

C Find the surface area of an object.

A Area

Area

Here is a formal definition of Area:

Definition

The **area** of a flat object is a measure of the amount of surface the object has.

Now let's examine the rectangle in Figure 1 to visualize square feet on a smaller scale.

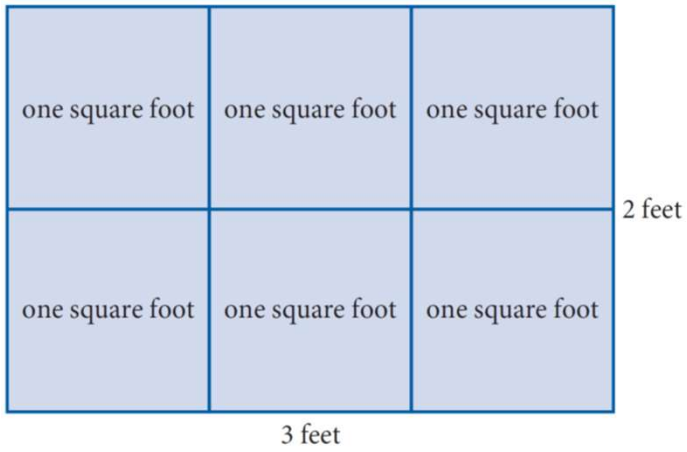


Figure 1

Area

The area of the given rectangle is 6 square feet, because it takes 6 square feet to cover it.

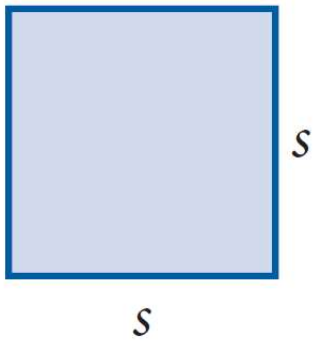
The area of this rectangle can be found by multiplying the length and the width.

$$\begin{aligned}\text{Area} &= (\text{length}) \cdot (\text{width}) \\ &= (3 \text{ feet}) \cdot (2 \text{ feet}) \\ &= (3 \cdot 2) \cdot (\text{feet} \cdot \text{feet}) \\ &= 6 \text{ square feet}\end{aligned}$$

Area

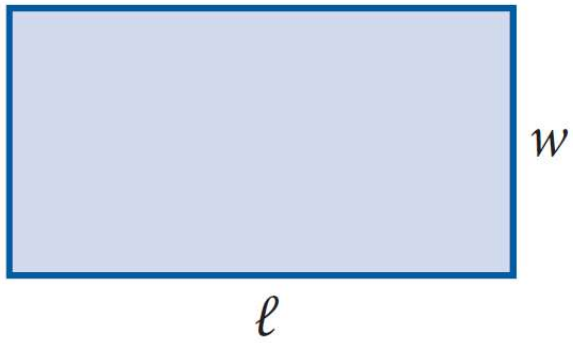
Here are three common geometric figures along with the formula for the area of each one.

Square



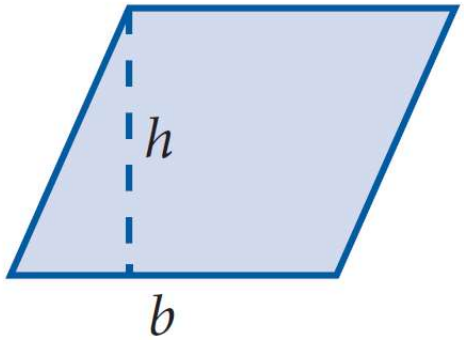
Area = (side)(side)
= $(\text{side})^2$
= s^2

Rectangle



Area = (length)(width)
$A = lw$

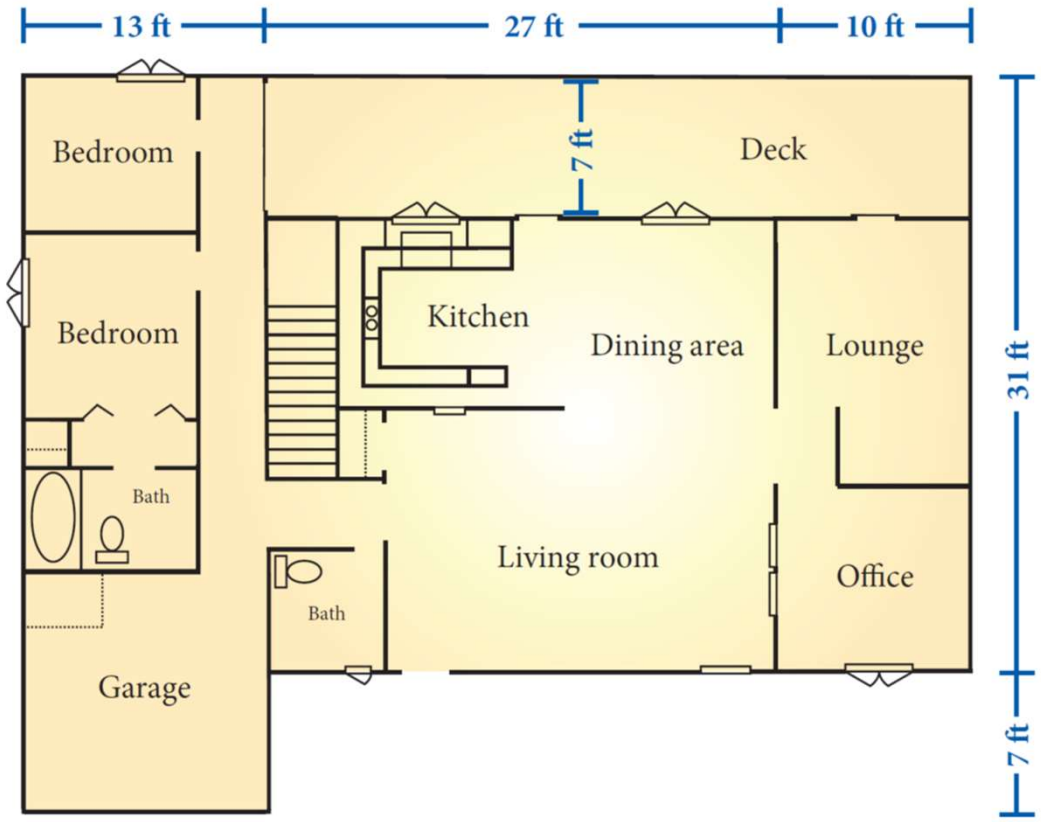
Parallelogram



Area = (base)(height)
$A = bh$

Example 3

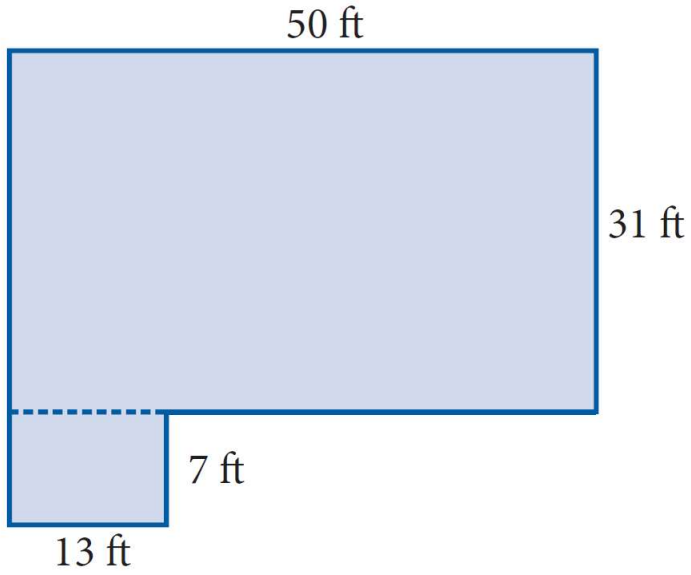
Find the total area of the house and garage shown below.



Example 3 – *Solution*

We begin by drawing an additional line, so that the original figure is now composed of two rectangles.

Next, we fill in the missing dimensions on the two rectangles.



Example 3 – *Solution*

cont'd

Finally, we calculate the area of the original figure by adding the areas of the individual figures.

$$\begin{aligned}
\text{Area} &= \text{Area of the small rectangle} + \text{Area of the large rectangle} \\
&= 13 \cdot 7 + 50 \cdot 31 \\
&= 91 + 1{,}550 \\
&= 1{,}641 \text{ square feet}
\end{aligned}$$

B Volume

Volume

Next, we move up one dimension and consider what is called *volume*. Here is a definition for volume:

Definition

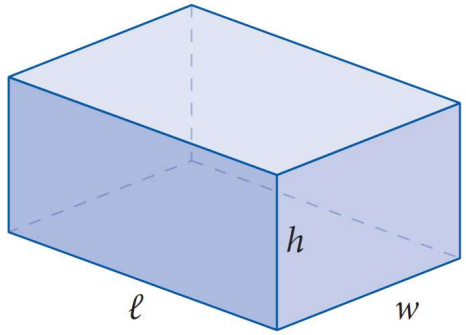
Volume is the measure of the space enclosed by a solid.

For the present, we will confine our discussion of volume to volumes of *rectangular solids*.

Volume

Rectangular solids are the three-dimensional equivalents of rectangles, where opposite sides are parallel, and any two sides that meet, meet at right angles.

A rectangular solid is shown in Figure 3, along with the formula used to calculate its volume.

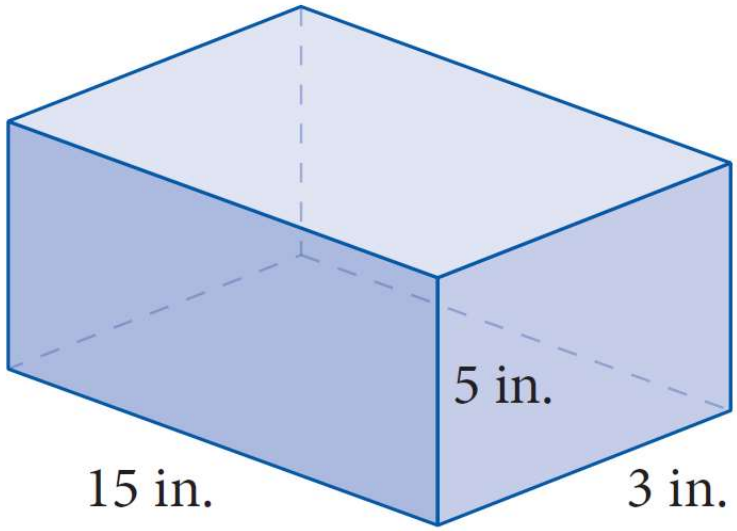


Volume = (length)(width)(height)
$V = \ell wh$

Figure 3

Example 4

Find the volume of a rectangular solid with length 15 inches, width 3 inches, and height 5 inches.



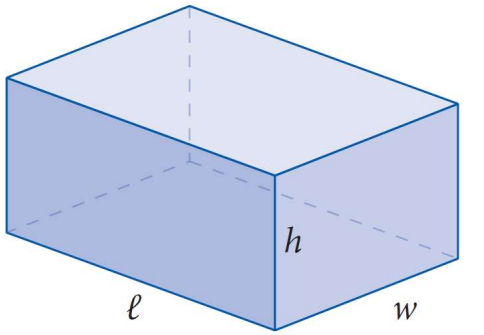
Example 4 – *Solution*

To find the volume, we apply the formula shown in Figure 3.

$$V = l \cdot w \cdot h$$

$$= (15 \text{ in.})(3 \text{ in.})(5 \text{ in.})$$

$$= 225 \text{ in}^3$$



Volume = (length)(width)(height)
$V = \ell wh$

Figure 3

C Surface Area

Surface Area

Now let's shift our discussion to *surface area*.

Definition

Surface area is the total area of all surfaces of an object.

Figure 4 shows a closed box with length *l*, width *w*, and height *h*.

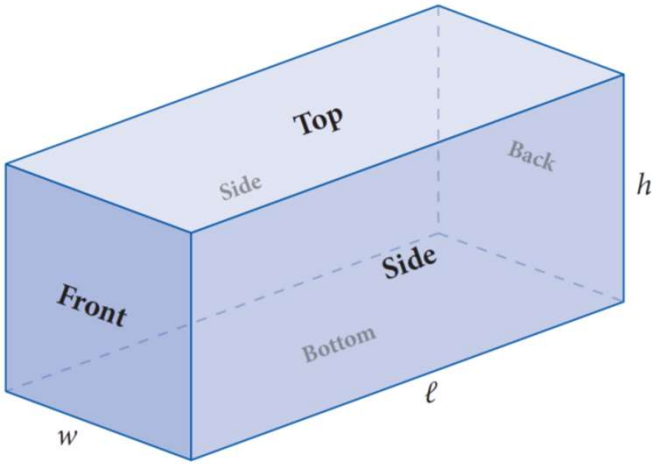


Figure 4

Surface Area

The surfaces of the box are labeled as sides, top, bottom, front, and back.

To find the surface area of the box, we add the areas of each of the six surfaces that are labeled in Figure 4.

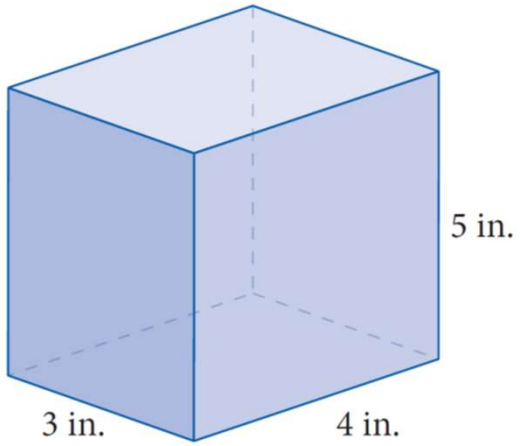
$$\text{Surface area} = \text{side} + \text{side} + \text{front} + \text{back} + \text{top} + \text{bottom}$$

$$S = (l \cdot h) + (l \cdot h) + (h \cdot w) + (h \cdot w) + (l \cdot w) + (l \cdot w)$$

$$= 2lh + 2hw + 2lw$$

Example 5

Find the surface area of the box below.



Solution:

To find the surface area, we find the area of each surface individually, and then we add them together.

Example 5 – *Solution*

$$\text{Surface area} = 2(3\text{ in.})(4\text{ in.}) + 2(3\text{ in.})(5\text{ in.}) + 2(4\text{ in.})(5\text{ in.})$$

$$= 24\text{ in}^2 + 30\text{ in}^2 + 40\text{ in}^2$$

$$= 94\text{ in}^2$$

The total surface area is 94 square inches. If we calculate the volume enclosed by the box, it is

$$V = (3\text{ in.})(4\text{ in.})(5\text{ in.}) = 60\text{ in}^3.$$

The surface area measures how much material it takes to make the box, whereas the volume measures how much space the box will hold.