Whole Numbers

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SECTION 1.7

Exponents, Order of Operations, and Averages

Objectives

A Identify the base and exponent of an expression.

B Simplify expressions with exponents.

C Use the rule for order of operations.

D Find the mean, median, mode, and range of a set of numbers.

A Exponents

Exponents

Exponents are a shorthand way of writing repeated multiplication.

In the expression 2^3, 2 is called the *base* and 3 is called the *exponent*.

The expression 2^3 is read "2 to the third power" or "2 cubed."

The exponent 3 tells us to use the base 2 as a multiplication factor three times.

$2^3 = 2 \cdot 2 \cdot 2$ 2 is used as a factor three times.

Exponents

We can simplify the expression by multiplication.

$$\begin{aligned} 2^3 &= 2 \cdot 2 \cdot 2 \\ &= 4 \cdot 2 \\ &= 8 \end{aligned}$$

The expression 2^3 is equal to the number 8.

Exponents

We can summarize this discussion with the following definition.

Definition

An **exponent** is a whole number that indicates how many times the base is to be used as a factor. Exponents indicate repeated multiplication.

Example 1

3^2 The base is 3, and the exponent is 2.

The expression is read “3 to the second power” or “3 squared.”

Exponents

A base raised to the second power is also said to be *squared*, and a base raised to the third power is also said to be *cubed*.

These are the only two exponents (^{2} and ^{3}) that have special names.

All other exponents are referred to only as “fourth powers,” “fifth powers,” “sixth powers,” and so on.

B Expressions with Exponents

Expression with Exponents

The next examples show how we can simplify expressions involving exponents by using repeated multiplication.

Examples

Example 4

$$3^2 = 3 \cdot 3$$

$$= 9$$

Example 5

$$4^2 = 4 \cdot 4$$

$$= 16$$

Expressions with Exponents

Finally, we should consider what happens when the numbers 0 and 1 are used as exponents.

First of all, let's look at the following rule where the number 1 appears as an exponent:

Rule **Exponent 1**

Any number raised to the first power is itself. That is, if we let the letter a represent any number, then

$$a^1 = a$$

Expressions with Exponents

To take care of the cases when 0 is used as an exponent, we must use the following rule:

Rule **Exponent 0**

Any number other than 0 raised to the 0 power is 1. That is, if a represents any nonzero number, then it is always true that

$$a^0 = 1$$

Examples

Example 9

$$5^1 = 5$$

Example 11

$$4^0 = 1$$

C Order of Operations

Order of Operations

The symbols we use to specify operations, +, –, ·, ÷, along with the symbols we use for grouping, ( ) and [ ], serve the same purpose in mathematics as punctuation marks in English. They may be called the punctuation marks of mathematics.

Consider the following sentence:

Bob said John is tall.

It can have two different meanings, depending on how we punctuate it:

1. “Bob,” said John, “is tall.”

2. Bob said, “John is tall.”

Order of Operations

Without the punctuation marks we don’t know which meaning the sentence has.

Now, consider the following mathematical expression:

$$4 + 5 \cdot 2$$

What should we do? Should we add 4 and 5 first, or should we multiply 5 and 2 first?

There seem to be two different answers.

Order of Operations

In mathematics we want to avoid situations in which two different results are possible. Therefore we follow the rule for order of operations.

Rule Order of Operations

When evaluating mathematical expressions, we will perform the operations in the following order:

1. If the expression contains grouping symbols, such as parentheses ( ), brackets [ ], or a fraction bar, then we perform the operations inside the grouping symbols, or above and below the fraction bar, first.
2. Then we evaluate, or simplify, any numbers with exponents.
3. Then we do all multiplications and divisions in order, starting at the left and moving right.
4. Finally, we do all additions and subtractions, from left to right.

Order of Operations

According to our rule, the expression $4 + 5 \cdot 2$ would have to be evaluated by multiplying 5 and 2 first, and then adding 4.

The correct answer—and the only answer—to this problem is 14.

$$
\begin{aligned}
4 + 5 \cdot 2 &= 4 + 10 \qquad \text{Multiply first, then add.} \\
&= 14
\end{aligned}
$$

Example 13

Simplify: $4 \cdot 8 - 2 \cdot 6$

Solution:

We multiply first and then subtract:

$$4 \cdot 8 - 2 \cdot 6 = 32 - 12$$

Multiply first, then subtract.

$$= 20$$

Order of Operations

Table 1 lists some English expressions and their corresponding mathematical expressions written in symbols.

TABLE 1

In English	Mathematical Equivalent
5 times the sum of 3 and 8	$5(3 + 8)$
Twice the difference of 4 and 3	$2(4 - 3)$
6 added to 7 times the sum of 5 and 6	$6 + 7(5 + 6)$
The sum of 4 times 5 and 8 times 9	$4 \cdot 5 + 8 \cdot 9$
3 subtracted from the quotient of 10 and 2	$10 \div 2 - 3$

D Average

Average

Next we turn our attention to averages. If we go online, we find the following definition for the word *average* when it is used as a noun:

Definition

Average (*noun*) is a single value (as a mean, mode, or median) that summarizes or represents the general significance of a set of unequal values.

Average

We will discuss all three individually now. First, lets define the most common average: *mean*.

Mean

Mean

Definition

To find the **mean** for a set of values, we add all the numbers and then divide the sum by the number of values in the set. The mean is sometimes called the **arithmetic mean**.

Median

Median

The median for a set of numbers is the number such that half of the numbers in the set are above it and half are below it.

Here is the exact definition:

Definition

To find the **median** for a set of values, we write the values in order from smallest to largest. If there is an odd number of values, the median is the middle value. If there is an even number of values, then the median is the mean of the two values in the middle.

Mode

Mode

When we have a set of values in which one value occurs more often than the rest, that value is the *mode*.

Definition

The **mode** for a set of values is the value that occurs most frequently. If all the values in the set occur the same number of times, there is no mode.

Example 20

A math class with 18 students had the grades shown below on their first test. Find the mean, the median, and the mode.

77	87	100	65	79	87
79	85	87	95	56	87
56	75	79	93	97	92

Solution:

To find the mean, we add all the scores and divide by 18:

$$\text{Mean} = \frac{77+87+100+65+79+87+79+85+87+95+56+87+56+75+79+93+97+92}{18}$$

Example 20 – *Solution*

cont'd

$$= \frac{1{,}476}{18}$$

$$= 82$$

To find the median, we must put the test scores in order from smallest to largest; then, because there are an even number of test scores, we must find the mean of the middle two scores.

56 56 65 75 77 79 79 79 85 87 87 87 87 92 93 95 97 100

$$\text{Median} = \frac{85 + 87}{2} = 86$$

Example 20 – *Solution*

cont'd

The mode is the most frequently occurring score. Because 87 occurs 4 times, and no other scores occur that many times, 87 is the mode.

The mean is 82, the median is 86, and the mode is 87.

More Vocabulary

More Vocabulary

Below is the definition of the word average when it is used as a verb.

Definition

Average (*verb*) is to find the arithmetic mean of a series of unequal quantities.

Range

Range

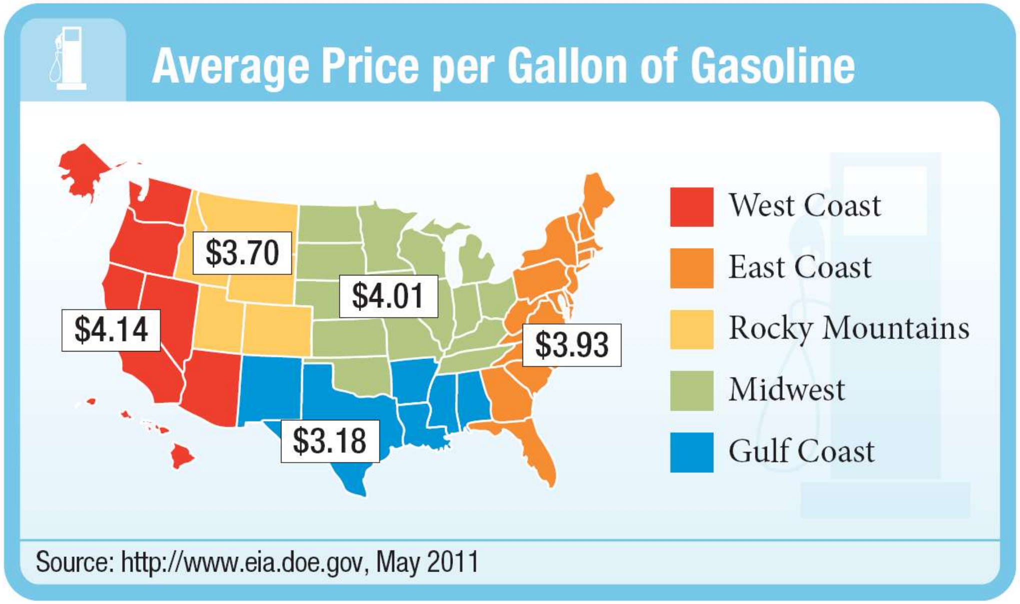
Another way to analyze a set of data is to find the range.

Definition

The **range** of a set of numbers is the difference between the greatest and least values.

Range

The following chart shows average gas prices around the country.



From the information above, we see that the lowest average price was found in the Gulf Coast at $3.18 per gallon.

Range

The highest price is on the West Coast at $4.14 per gallon. The range of this set of data is the difference between these two numbers:

$$\$4.14 - \$3.18 = \$0.96$$

We say the country’s average gas prices had a range of $0.96.