Whole Numbers

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SECTION 1.6

Division with Whole Numbers

Objectives

A Understand the notation and vocabulary of division.

B Divide whole numbers.

C Solve applications using division.

Division with Whole Numbers

Suppose a sporting goods store purchases 500 basketballs for a total wholesale cost of $2,500.

In order to calculate the price of each basketball, we need to use division: dividing 2500 by 500.

As a division problem:	As a multiplication problem:
$2500 \div 500 = 5$	$5 \cdot 500 = 2500$

As you can see, this problem can be thought of in terms of division, as well as in terms of multiplication. You will see later in this section why this is true.

A Notation

Notation

As was the case with multiplication, there are many ways to indicate division.

All the following statements are equivalent.

They all mean 10 divided by 5.

$$10 \div 5, \quad \frac{10}{5}, \quad 10/5, \quad 5\overline{)10}$$

The kind of notation we use to write division problems will depend on the situation.

Notation

We will use the notation $5\overline{)10}$ mostly with the long-division problems found in this chapter.

The notation $\frac{10}{5}$ will be used in the chapter on fractions and in later chapters.

The horizontal line used with the notation $\frac{10}{5}$ is called the *fraction bar*.

Vocabulary

Vocabulary

The word *quotient* is used to indicate division.

If we say "The quotient of 10 and 5 is 2," then we mean

$$10 \div 5 = 2 \qquad \text{or} \qquad \frac{10}{5} = 2$$

The 10 is called the *dividend*, and the 5 is called the *divisor*, and the 2 is called the *quotient*.

This quotient is a result of dividing 10 by 5 using the expressions $10 \div 5$ or $\frac{10}{5}$.

Vocabulary

TABLE 1

In English	In Symbols
The quotient of 15 and 3	$15 \div 3$, or $\frac{15}{3}$, or $15/3$
The quotient of 3 and 15	$3 \div 15$, or $\frac{3}{15}$, or $3/15$
The quotient of 8 and n	$8 \div n$, or $\frac{8}{n}$, or $8/n$
x divided by 2	$x \div 2$, or $\frac{x}{2}$, or $x/2$
The quotient of 21 and 3 is 7.	$21 \div 3 = 7$, or $\frac{21}{3} = 7$

The Meaning of Division

The Meaning of Division

One way to arrive at an answer to a division problem is by thinking in terms of multiplication.

For example, if we want to find the quotient of 32 and 8, we may ask, "What do we multiply by 8 to get 32?"

$$32 \div 8 = ? \quad \text{means} \quad 8 \cdot ? = 32$$

Because we know from our work with multiplication that $8 \cdot 4 = 32$, it must be true that

$$32 \div 8 = 4$$

The Meaning of Division

Table 2 lists some additional examples.

TABLE 2

Division		Multiplication
$18 \div 6 = 3$	because	$6 \cdot 3 = 18$
$32 \div 8 = 4$	because	$8 \cdot 4 = 32$
$10 \div 2 = 5$	because	$2 \cdot 5 = 10$
$72 \div 9 = 8$	because	$9 \cdot 8 = 72$

B Division by One-Digit Numbers

Division by One-Digit Numbers

Consider the following division problem:

$$465 \div 5$$

We can think of this problem as asking the question, "How many fives can we subtract from 465?"

To answer the question we begin subtracting multiples of 5.

Here is one way to organize this process:

$$\begin{array}{r l}
90 & \longleftarrow \text{We first guess that there are at least 90 fives in 465.} \\
5\overline{)465} & \\
-\ 450 & \longleftarrow 90(5) = 450 \\
\hline
15 & \longleftarrow \text{15 is left after we subtract 90 fives from 465.}
\end{array}$$

Division by One-Digit Numbers

What we have done so far is subtract 90 fives from 465 and found that 15 is still left.

Because there are 3 fives in 15, we continue the process.

$$\begin{array}{r l}
3 & \longleftarrow \text{There are 3 fives in 15.} \\
90 & \\
5\overline{)465} & \\
-450 & \\
\cline{1-1}
15 & \\
-\ 15 & \longleftarrow 3 \cdot 5 = 15 \\
\cline{1-1}
0 & \longleftarrow \text{The difference is 0.}
\end{array}$$

The total number of fives we have subtracted from 465 is

$$90 + 3 = 93$$

Division by One-Digit Numbers

We now summarize the results of our work.

$465 \div 5 = 93$ We check our answer with multiplication. $\longrightarrow$

$$\begin{array}{r} \overset{1}{9}3 \\ \times\;\; 5 \\ \hline 465 \end{array}$$

The division problem just shown can be shortened by eliminating the subtraction signs, eliminating the zeros in each estimate, and eliminating some of the numbers that are repeated in the problem.

Division by One-Digit Numbers

	3			
	90		93	
The shorthand	$5\overline{)465}$	looks like	$5\overline{)465}$	The arrow
form for this	$\underline{450}$	this:	$\underline{45}\downarrow$	indicates that
problem ⟶	15		15	we bring down
	$\underline{15}$		$\underline{15}$	the 5 after
	0		0	we subtract.

The problem shown above on the right is the shortcut form of what is called *long division*.

The next example shows this shortcut form of long division from start to finish.

Example 1

Divide: 595 ÷ 7

Solution:

Because 7(8) = 56, our first estimate of the number of sevens that can be subtracted from 595 is 80:

$$
\begin{array}{r l}
8 & \longleftarrow \text{The 8 is placed above the tens column} \\
7\overline{)595} & \ \text{so we know our first estimate is 80.} \\
\underline{56}\downarrow & \longleftarrow 8(7) = 56 \\
35 & \longleftarrow 59 - 56 = 3\text{; then bring down the 5.}
\end{array}
$$

Example 1 – *Solution*

cont'd

Since 7(5) = 35, we have

$$
\begin{array}{r l}
85 & \longleftarrow \text{There are 5 sevens in 35.} \\
7\overline{)595} & \\
\underline{56}\downarrow & \\
35 & \\
\underline{35} & \longleftarrow 5(7) = 35 \\
0 & \longleftarrow 35 - 35 = 0
\end{array}
$$

Our result is 595 ÷ 7 = 85, which we can check with multiplication:

$$
\begin{array}{r}
\overset{3}{8}5 \\
\underline{\times\ 7} \\
595
\end{array}
$$

Division by Two-Digit Numbers

Example 2

Divide: 9,380 ÷ 35

Solution:

In this case our divisor, 35, is a two-digit number. The process of division is the same.

We still want to find the number of thirty-fives we can subtract from 9,380.

$$
\begin{array}{rl}
2 & \longleftarrow \text{The 2 is placed above the hundreds column.} \\
35\overline{)9380} & \\
\underline{70}\downarrow & \longleftarrow 2(35) = 70 \\
238 & \longleftarrow 93 - 70 = 23\text{; then bring down the 8.}
\end{array}
$$

Example 2 – *Solution*

cont'd

We can make a few preliminary calculations to help estimate how many thirty-fives are in 238:

$5 \times 35 = 175$ $6 \times 35 = 210$ $7 \times 35 = 245$

Because 210 is the closest to 238 without being larger than 238, we use 6 as our next estimate:

$$\begin{array}{r l}
26 & \longleftarrow \text{6 in the tens column means this estimate is 60.} \\
35\overline{)9380} & \\
\underline{70} & \\
238 & \\
\underline{210} & \longleftarrow 6(35) = 210 \\
280 & \longleftarrow 238 - 210 = 28\text{; bring down the 0.}
\end{array}$$

Example 2 – *Solution*

Because 35(8) = 280, we have

$$
\begin{array}{r}
268 \\
35\overline{)9380} \\
\underline{70} \\
238 \\
\underline{210} \\
280 \\
\underline{280} \\
0
\end{array}
\qquad
\begin{array}{l}
\\ \\ \\ \\ \\ \\
\longleftarrow 8(35) = 280 \\
\longleftarrow 280 - 280 = 0
\end{array}
$$

Example 2 – *Solution*

cont'd

We can check our result with multiplication:

$$\begin{array}{r} 268 \\ \times \quad 35 \\ \hline 1{,}340 \\ 8{,}040 \\ \hline 9{,}380 \end{array}$$

Division with Remainders

Division with remainders

Darlene is planning to serve 6-ounce glasses of soda at a party.

To see how many glasses she could fill from the 32-ounce bottle, she would divide 32 by 6.

If she did so, she would find that she could fill 5 glasses, but after doing so she would have 2 ounces of soda left in the bottle.

Division with remainders

The 2 ounces is known as the remainder. A diagram of this problem is shown in Figure 1.

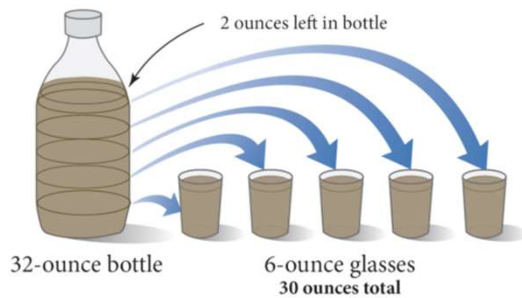

Figure 1

Writing the results in the diagram as a division problem looks like this:

$$\begin{array}{rr l}
 & 5 & \longleftarrow \text{Quotient} \\
\text{Divisor} \longrightarrow & 6\overline{)32} & \longleftarrow \text{Dividend} \\
 & \underline{30} & \\
 & 2 & \longleftarrow \text{Remainder}
\end{array}$$

Example 4

Divide: 1,690 ÷ 67

Solution:

Dividing as we have previously, we get

$$
\begin{array}{rl}
25 & \\
67\overline{)1690} & \\
\underline{134} & \\
350 & \\
\underline{335} & \\
15 & \longleftarrow \text{15 is left over.}
\end{array}
$$

We have 15 left, and because 15 is less than 67, no more sixty-sevens can be subtracted.

Example 4 – *Solution*

cont'd

In a situation like this we call 15 the *remainder* and write

These both indicate that the remainder is 15.

$$\begin{array}{r} 25 \text{ R } 15 \\ 67\overline{)1690} \\ \underline{134} \\ 350 \\ \underline{335} \\ 15 \end{array} \qquad \text{or} \qquad \begin{array}{r} 25\frac{15}{67} \\ 67\overline{)1690} \\ \underline{134} \\ 350 \\ \underline{335} \\ 15 \end{array}$$

Both forms of notation shown above indicate that 15 is the remainder.

The notation R 15 is the notation we will use in this chapter.

Example 4 – *Solution*

cont'd

The notation $\frac{15}{67}$ will be useful in the chapter on fractions.

To check a problem like this, we multiply the divisor and the quotient as usual, and then add the remainder to this result:

$$\begin{array}{r} 67 \\ \times\ 25 \\ \hline 335 \\ 1{,}340 \\ \hline 1{,}675 \end{array} \longleftarrow \text{Product of divisor and quotient}$$

$$\underset{\substack{\uparrow \\ \text{Remainder}}}{1{,}675 + 15} = \underset{\substack{\nwarrow \\ \text{Dividend}}}{1{,}690}$$

C Applications

Example 5

A family has an annual income of $35,880. How much is their average monthly income?

Solution:

Because there are 12 months in a year and the yearly (annual) income is $35,880, we want to know what $35,880 divided into 12 equal parts is.

Example 5 – *Solution*

cont'd

Therefore we have

$$
\begin{array}{r}
2990 \\
12\overline{)35880} \\
\underline{24} \\
118 \\
\underline{108} \\
108 \\
\underline{108} \\
00
\end{array}
$$

Because 35,880 ÷ 12 = 2,990, the monthly income for this family is $2,990.

Division by Zero

Division by Zero

We cannot divide by 0. That is, we cannot use 0 as a divisor in any division problem.

Suppose there was an answer to the problem

$$\frac{8}{0} = ?$$

That would mean that

$$0 \cdot ? = 8$$

But, because of the multiplication property of zero, we know that multiplication by 0 always produces 0.

There is no number we can use for the ? to make a true statement out of $0 \cdot ? = 8$

Division by Zero

Because this was equivalent to the original division problem

$$\frac{8}{0} = ?$$

we have no number to associate with the expression $\frac{8}{0}$. It is undefined. Here is the formal rule:

Rule **Division by Zero**

Division by 0 is undefined. Any expression with a divisor of 0 is undefined. We cannot divide by 0.