Whole Numbers

1

SECTION 1.5

Multiplication with Whole Numbers

Objectives

A Multiply whole numbers using repeated addition.

B Understand the notation and vocabulary of multiplication.

C Identify properties of multiplication.

D Solve equations with multiplication.

E Solve applications with multiplication.

A Multiplying Whole Numbers

Multiplying Whole Numbers

To begin, we can think of multiplication as shorthand for repeated addition.

That is, multiplying 3 times 4 can be thought of this way:

$$3 \text{ times } 4 = 4 + 4 + 4 = 12$$

Multiplying 3 times 4 means to add three 4's. We can write 3 times 4 as 3×4, or $3 \cdot 4$.

Example 1

Multiply: $3 \cdot 4{,}000$

Solution:

Using the definition of multiplication as repeated addition, we have

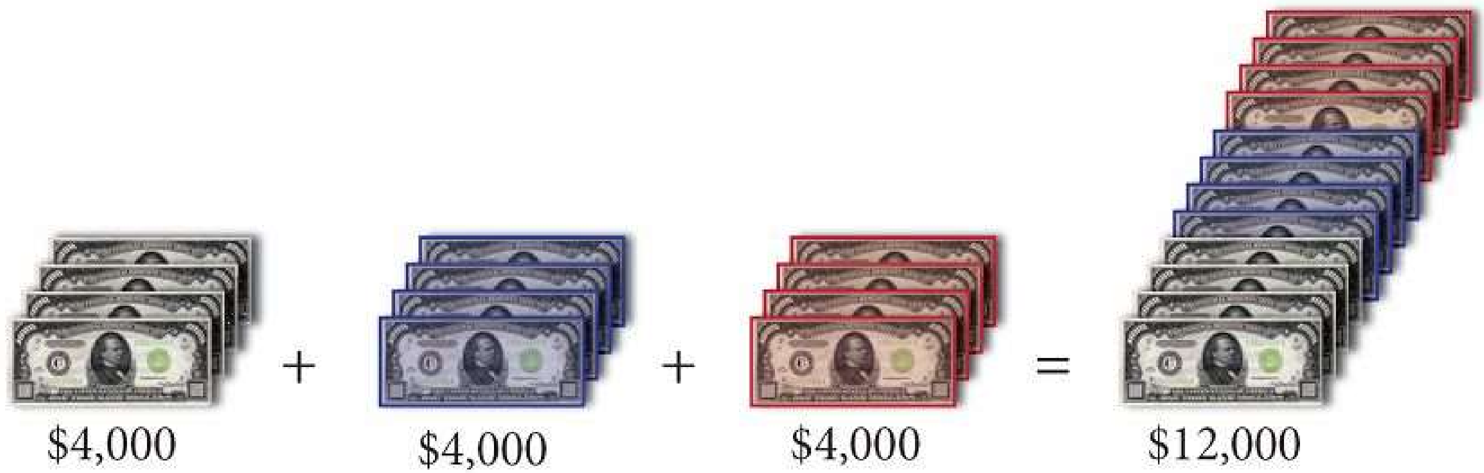
$$3 \cdot 4{,}000 = 4{,}000 + 4{,}000 + 4{,}000$$

$$= 12{,}000$$

Example 1 – *Solution*

cont'd

Here is one way to visualize this process.



Notice that if we had multiplied 3 and 4 to get 12 and then attached three zeros on the right, the result would have been the same.

Facts of Multiplication

Facts of Multiplication

Here are the basic multiplication facts that will help you in this section.

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

B Notation and Vocabulary

Notation and Vocabulary

There are many ways to indicate multiplication. All the following statements are equivalent.

They all indicate multiplication with the numbers 3 and 4.

$$3 \cdot 4, \quad 3 \times 4, \quad 3(4), \quad (3)4, \quad (3)(4), \quad \begin{array}{r} 4 \\ \times\, 3 \\ \hline \end{array}$$

If one or both of the numbers we are multiplying are represented by letters, we may also use the following notation:

$5n$	means	5 times n
ab	means	a times b

Notation and Vocabulary

We use the word *product* to indicate multiplication. If we say, "The product of 3 and 4 is 12," then we mean

$$3 \cdot 4 = 12$$

Both $3 \cdot 4$ and 12 are called the product of 3 and 4. The 3 and 4 are called *factors*.

Definition

Factors are numbers that, when multiplied together, give a product.

Notation and Vocabulary

Table 1 gives some word statements involving multiplication and their mathematical equivalents written in symbols.

TABLE 1

In English	In Symbols
The product of 2 and 5	$2 \cdot 5$
The product of 5 and 2	$5 \cdot 2$
The product of 4 and n	$4n$
The product of x and y	xy
The product of 9 and 6 is 54.	$9 \cdot 6 = 54$
The product of 2 and 8 is 16.	$2 \cdot 8 = 16$

Example 2

Identify the products and factors in the statement

$$9 \cdot 8 = 72$$

Solution:

The factors are 9 and 8, and the products are $9 \cdot 8$ and 72.

C Properties of Multiplication

Properties of Multiplication

To develop an efficient method of multiplication, we need to use what is called the *distributive property*.

To begin, consider the following two problems:

Problem 1	Problem 2
$3(4 + 5)$	$3(4) + 3(5)$
$= 3(9)$	$= 12 + 15$
$= 27$	$= 27$

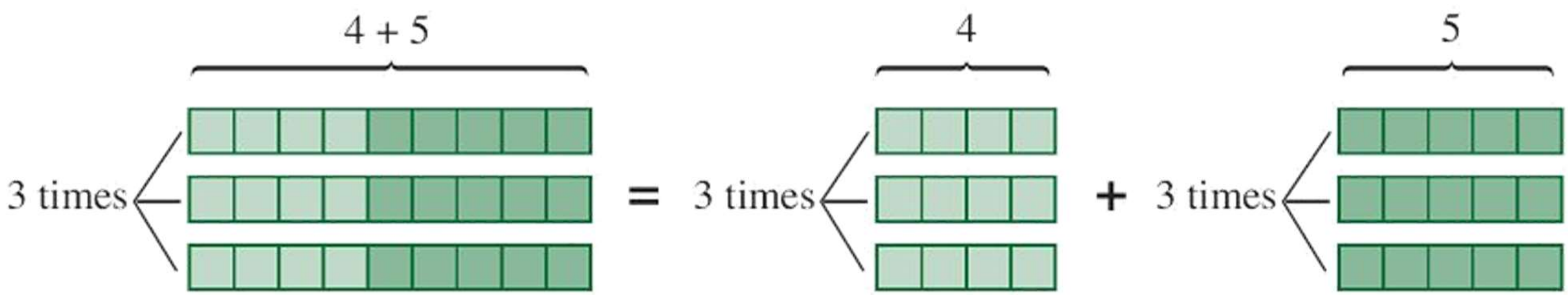
The result in both cases is the same number, 27. This indicates that the original two expressions must have been equal also. That is,

$$3(4 + 5) = 3(4) + 3(5)$$

Properties of Multiplication

This is an example of the distributive property. We say that multiplication *distributes* over addition.

$$3(4 + 5) = 3(4) + 3(5)$$



$$3(4 + 5) = 3 \cdot 4 + 3 \cdot 5$$

Properties of Multiplication

We can write this property in symbols using the letters *a*, *b*, and *c* to represent any three whole numbers.

Distributive Property

If a, b, and c represent any three whole numbers, then

$$a(b + c) = a(b) + a(c)$$

Properties of Multiplication

Suppose we want to find the product 7(65).

By writing 65 as 60 + 5 and applying the distributive property, we have:

$7(65)$	$= 7(60 + 5)$	$65 = 60 + 5$
	$= 7(60) + 7(5)$	Distributive property
	$= 420 + 35$	Multiply.
	$= 455$	Add.

Properties of Multiplication

We can write the same problem vertically like this:

$$\begin{array}{r@{\quad}l}
60 + 5 & \\
\times \qquad 7 & \\
\hline
35 & \leftarrow \quad 7(5) = 35 \\
+ \quad 420 & \leftarrow \quad 7(60) = 420 \\
\hline
455 &
\end{array}$$

This saves some space in writing.

Properties of Multiplication

Notice that we can cut down on the amount of writing even more if we write the problem this way:

STEP 1: 7(5) = 35; write the 5 in the ones column, and then carry the 3 to the tens column.

STEP 2: 7(6) = 42; add the 3 we carried to 42 to get 45.

$$\begin{array}{r} {}^{3}\\ 65 \\ \times\ 7 \\ \hline 455 \end{array}$$

This shortcut notation takes some practice.

Example 4

Multiply: 9(43).

STEP 2: $9(4) = 36$; add the 2 we carried to 36 to get 38.

$$\begin{array}{r} \scriptstyle 2 \\ 43 \\ \times\ \ 9 \\ \hline 387 \end{array}$$

STEP 1: $9(3) = 27$; write the 7 in the ones column, and then carry the 2 to the tens column.

Properties of Multiplication

Here are some other important properties of multiplication:

Multiplication Property of 0

If a represents any number, then

$$a \cdot 0 = 0 \qquad \text{and} \qquad 0 \cdot a = 0$$

In words: Multiplication by 0 always results in 0.

Multiplication Property of 1

If a represents any number, then

$$a \cdot 1 = a \qquad \text{and} \qquad 1 \cdot a = a$$

In words: Multiplying any number by 1 leaves that number unchanged.

Properties of Multiplication

Commutative Property of Multiplication

If a and b are any two numbers, then

$$ab = ba$$

In words: The order of the numbers in a product doesn't affect the result.

Associative Property of Multiplication

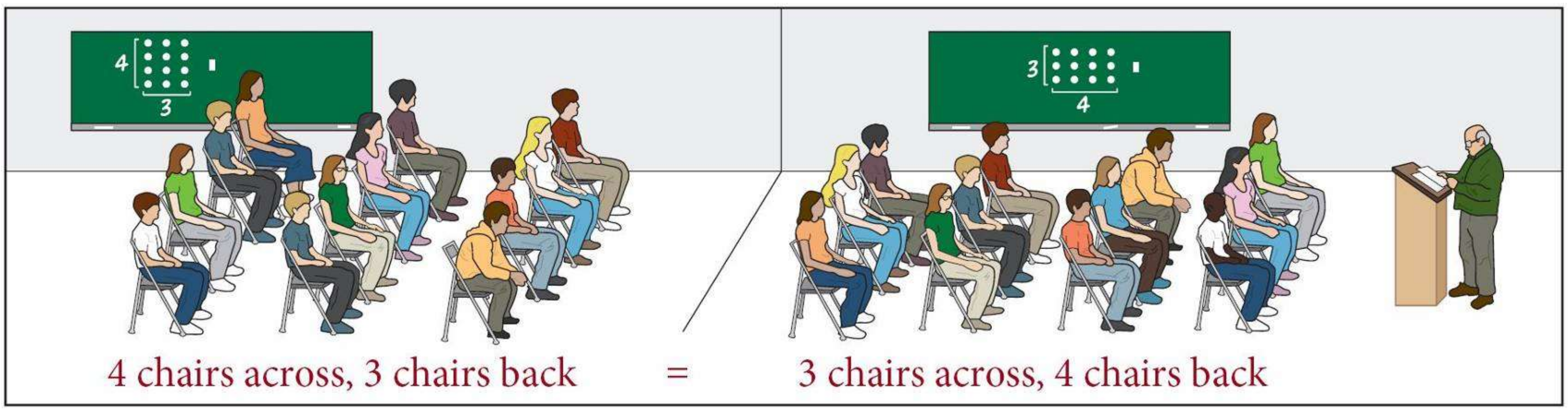
If a, b, and c represent any three numbers, then

$$(ab)c = a(bc)$$

In words: We can change the grouping of the numbers in a product without changing the result.

Properties of Multiplication

To visualize the commutative property, we can think of an instructor with 12 students.



Example 7

Use the commutative property of multiplication to rewrite each of the following products:

a. $7 \cdot 9$ **b.** $4(6)$

Solution:

Applying the commutative property to each expression, we have:

a. $7 \cdot 9 = 9 \cdot 7$

b. $4(6) = 6(4)$

D Solving Equations

Solving Equations

If n is used to represent a number, then the equation

$$4 \cdot n = 12$$

is read "4 times n is 12," or "The product of 4 and n is 12."

This means that we are looking for the number we multiply by 4 to get 12.

The number is 3. Because the equation becomes a true statement if n is 3, we say that 3 is the solution to the equation.

Example 9

Find the solution to each of the following equations:

a. $6 \cdot n = 24$ **b.** $4 \cdot n = 36$ **c.** $15 = 3 \cdot n$ **d.** $21 = 3 \cdot n$

Solution:

a. The solution to $6 \cdot n = 24$ is 4, because $6 \cdot 4 = 24$.

b. The solution to $4 \cdot n = 36$ is 9, because $4 \cdot 9 = 36$.

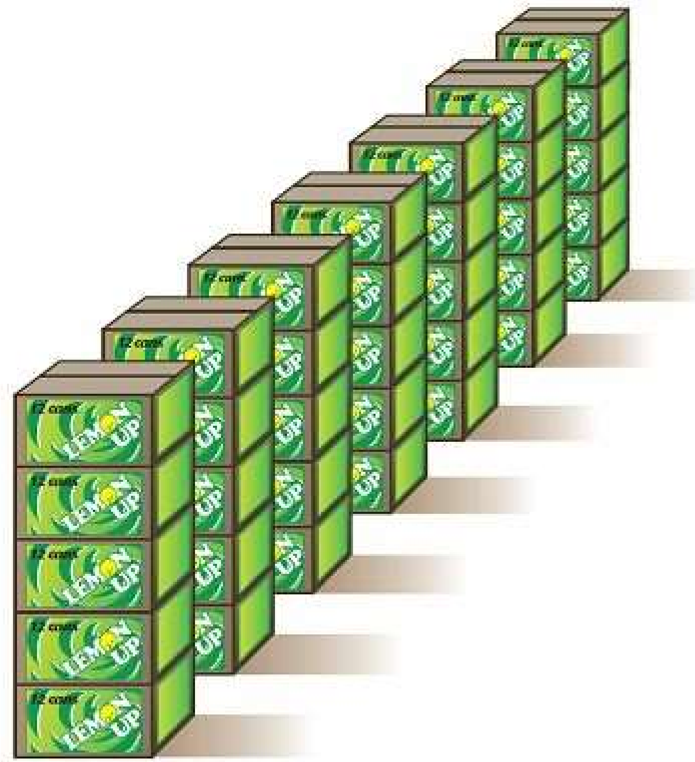
c. The solution to $15 = 3 \cdot n$ is 5, because $15 = 3 \cdot 5$.

d. The solution to $21 = 3 \cdot n$ is 7, because $21 = 3 \cdot 7$.

E Applications

Example 10

A supermarket orders 35 cases of a certain soft drink. If each case contains 12 cans of the drink, how many cans were ordered?



Example 10 – *Solution*

We have 35 cases, and each case has 12 cans.

The total number of cans is the product of 35 and 12, which is 35(12):

$$\begin{array}{r} 12 \\ \times \quad 35 \\ \hline 60 \\ +\ 360 \\ \hline 420 \end{array} \qquad \begin{array}{l} \\ \\ \longleftarrow \quad 5(12) = 60 \\ \longleftarrow \quad 30(12) = 360 \\ \\ \end{array}$$

There is a total of 420 cans of the soft drink.